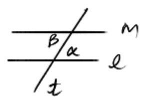
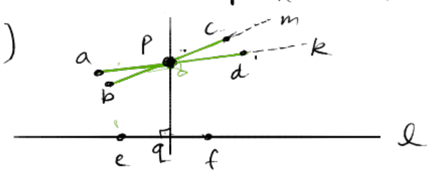
AIA : 

If a transversal cuts m & l s.t. alt. int. angles are $\cong \Rightarrow l \parallel m$

Converse to AIA : If $l \parallel m \Rightarrow$ AIA's are $\cong$.

Prop. 4.8 : Converse to AIA $\Leftrightarrow$ Hilbert's Euclid. $\parallel$ Postulate (given line l, point p, $\exists$ at most 1 line thru p $\parallel$ to l.)

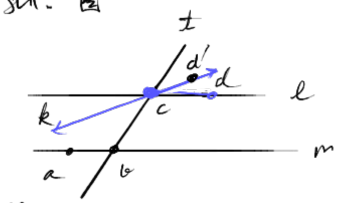


($\Rightarrow$) Assume Converse to AIA, show H.E. $\parallel$ P. holds. (at most 1 $\parallel$)

Let line l, point p be given. Assume lines m, k are s.t. $m \parallel l$ & $k \parallel l$.

Drop $\perp$ from p. Assumption $\Rightarrow$ AIA's are $\cong$. So

$\angle cpq \cong \angle pqe \cong \angle dpq$. If $d \in \overrightarrow{pc}$ $k = m$ & we're done. Else, assume $d \in$ Interior of $\angle cpq$. this implies $\angle cpq > \angle dpq$, impossible since they're both right. $\blacksquare$



($\Leftarrow$) Assume H.E. $\parallel$ P., show "If $\parallel \Rightarrow$ AIA's $\cong$".

Assume $m \parallel l$, cut by line n

If $\angle dcb \not\cong \angle abc$ then one is smaller. Choose it, say $\angle dcb$. Enlarge to $\angle d'cb$ then line $\overleftrightarrow{cd'}$ is a line thru p. this line is cut by t, as is m. The AIA's created are $\cong$ & by AIA Thm $\overleftrightarrow{cd'} \parallel m$

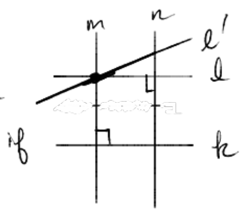
But this contradicts H.E. $\parallel$ P.

Prop 4.10: $k \parallel l \quad m \perp k \quad n \perp l \Rightarrow$ either $m = n$ or $m \parallel n$

$\Longleftrightarrow$

H.E.∥P.

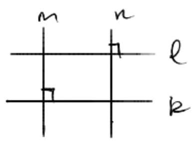
(⇒) Assume



if ... then $m = n$ or $m \parallel n$.

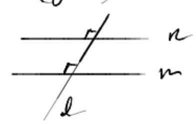
Assume l, l' are both $\parallel$ to k thru P. Let $m \perp l'$ & $n \perp l$, then by assumption $l = l'$ or $l \parallel l'$. Since $l \cap l' = P$ $\Rightarrow l = l'$ which proves H.E.∥P.

(⇐) If H.E.∥P. Assume $k \parallel l$, $m \perp k$, $n \perp l$. Show $m = n$ or $m \parallel n$.



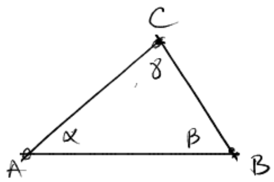
Assume $n \neq m$.

$k \parallel l$ & they are cut by transversals n, m making $90°$ w/ l, k respectively.
— by (4.8) $\Rightarrow$ $n \perp k$ & $m \perp l$. So $n \perp l$ & $m \perp l$. Now n, m are transversals cut by l.
(given)

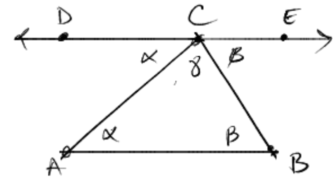


Since vertical angles are $\cong$ we the AIA theorem says $m \parallel n$.

4.11 H.E. ∥P ⇒ Angle Sum of $\Delta = 180°$



Proof: Lay off angle α @ C on segment CA
then $\overline{DC} \parallel \overline{AB}$ by AIA.
By H.E.∥P we know
converse to AIA so
$\angle ECB \cong \beta$



Finally $\alpha + \gamma + \beta = 180$ as the three angles form $\overline{DE}$

H.E∥P ⇒
Cor E.A of Δ = Sum of Remote Angles

$B + \theta = 180$ supplementary
$B + (\gamma + \alpha) = 180$
so $\theta = \gamma + \alpha$