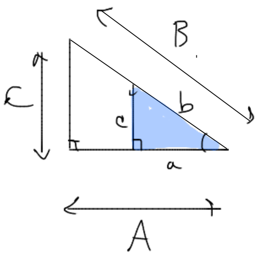
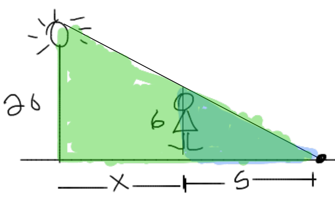
A street light is at the top of a 20 foot tall pole. A 6 foot tall woman walks away from the pole with a speed of 5 ft/sec along a straight path. How fast is the tip of her shadow moving when she is 40 feet from the base of the pole?

The tip of the shadow is moving at [ ] ft/sec.

Similar Triangles $\Rightarrow$ when $\underline{\text{all}}$ three angles of two triangles are the same then the ratio of corresponding sides is the same



$\Rightarrow \frac{a}{c} = \frac{A}{C}$



set $x = \text{dist}(\text{woman, light pole})$, so $\frac{dx}{dt} = 5$ ft/sec

$s =$ shadow length

tip of shadow moves @ $\frac{dx}{dt} + \frac{ds}{dt}$

Goal: $\frac{dx}{dt} + \frac{ds}{dt} = 5 + \frac{ds}{dt} = 5 + \frac{30}{14} \approx 7$ ft/s

Relate Var:

$\frac{20}{6} = \frac{x+s}{s}$

$20s = 6x + 6s$

$14s = 6x$

$s = \frac{6}{14}x$, apply $\frac{d}{dt}$

$\frac{ds}{dt} = \frac{6}{14}\overbrace{\frac{dx}{dt}}^{=5} = \frac{30}{14}$

#7

A landscape architect wished to enclose a rectangular garden on one side by a brick wall costing \$30/ft and on the other three sides by a metal fence costing \$20/ft. If the area of the garden is 8 square feet, find the dimensions of the garden that minimize the cost.

Length of side with bricks $x =$

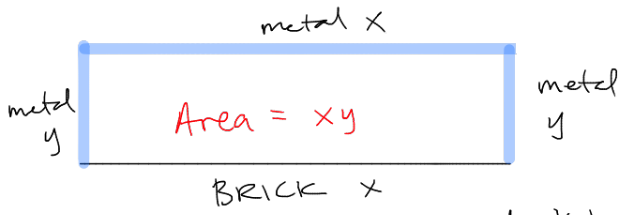
Length of adjacent side $y =$

($x, y =$ height " length

② create variables

③ create cost function

① visual



x

Cost = Brick price × amount + metal price × amount

$= 30\,\$/\text{ft} \cdot x\,\text{ft} + 20\,\$/\text{ft} \cdot (2y + x)\,\text{ft}$

$\text{Cost} = 30x + 20(2y + x)$ \$

since $8 = xy$, $y = \frac{8}{x}$

$C(x) = 30x + 20\left(\frac{16}{x} + x\right)$

$C(x) = 30x + \frac{320}{x} + 20x = 50x + \frac{320}{x}$

④ Minimize Cost ($\Rightarrow$ take deriv, set $= 0$ solve)

$c'(x) = 50 - \frac{320}{x^2} = 0$

$50 = \frac{320}{x^2} \Rightarrow 5x^2 = 32$

$x^2 = \frac{32}{5} \Rightarrow x = \sqrt{\frac{32}{5}} \approx 2.53$ ft

Find $y = \frac{8}{2.53} = 3.16$ ft

$V = \frac{4}{3}\pi r^3$

$A = 4\pi r^2$

spherical

Know: r = radius
D = diameter $= 2\cdot r$

dA/dt

derivative of area function is $= -1 \ \frac{cm^2}{min}$

Exam 4 Review

1. If a snowball melts so that its surface area decreases at a rate of 1 square centimeter per minute, find the rate at which the diameter decreases when the diameter is 10 centimaters.

Goal: $\frac{dD}{dt}$ when $D = 10$,

1 Strategy: get area formula $A = 4\pi r^2$

take deriv 2 $\frac{dA}{dt} = 8\pi r \cdot \frac{dr}{dt} = -1$

4 $\Rightarrow \boxed{\frac{dr}{dt} = \frac{-1}{8\pi\cdot 5} = \frac{-1}{40\pi} \ \frac{cm}{min}}$

Need: r; when 3 $D = 10$, $\boxed{r = 5}$

$\frac{dr}{dt}$, set $\frac{dA}{dt} = -1$ solve

5 So $D = 2r \Rightarrow \frac{dD}{dt} = 2\frac{dr}{dt} = 2\left(\frac{-1}{40\pi}\right)$

$= \frac{-1}{20\pi} \ \frac{cm}{min}$

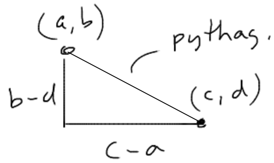
2. Two sides of a triangle are 4 m and 5 m in length and the angle between them is increasing at a rate of 0.06 radians per second. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is $\pi/3$.

Find the point P on the graph of the function $y = \sqrt{x}$ closest to the point $(7, 0)$

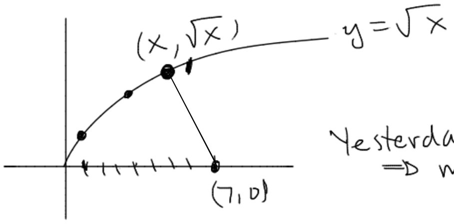
The x coordinate of P is: ______ .

$\Downarrow$

Find distance function & minimize it
(take derivative, set $= 0$)
solve



$$d = \sqrt{(a-c)^2 + (b-d)^2}$$



$$d = \sqrt{(x-7)^2 + (\sqrt{x} - 0)^2} = \sqrt{(x-7)^2 + x}$$

Yesterday
$\Rightarrow$ minimizing square root functn ≈ minimize square

$$D = d^2 = (x-7)^2 + x$$

$$\frac{dD}{dx} = 2(x-7) + 1 = 2x - 13 = 0$$

$$2x = 13 \quad , \quad x = \frac{13}{2} = 6.5$$

BTW the min dist is
plug $x = 6.5$ into $d = \sqrt{(6.5-7)^2 + 6.5} =$ ______