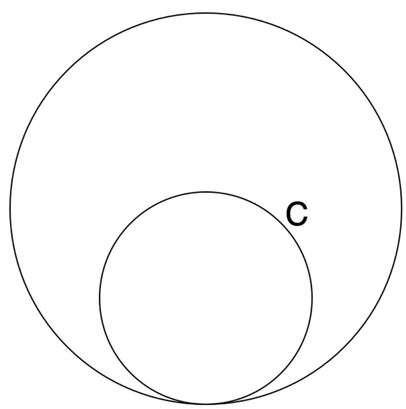
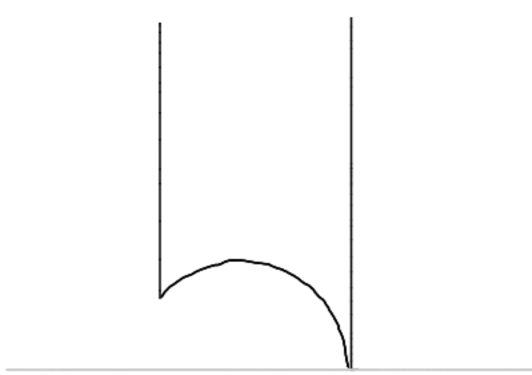
MA541 Final

1. Compute length of a horizontal curve in the Upper Half Plane model.

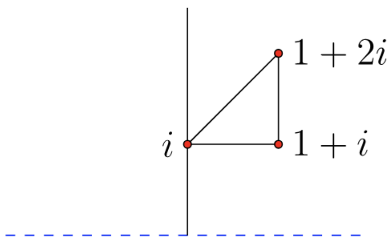
2. What becomes of a horocycle (like the one C pictured below) when we transfer the Poincare disk model of hyperbolic geometry to the upper half-plane model?



3. Give an explicit description of a transformation that takes an arbitrary double-ideal triangle in the upper half-plane to one with ideal points 1 and and an interior vertex on the upper half of the unit circle. Express the transformation as a Möbius Transformation and give its geometric classification.



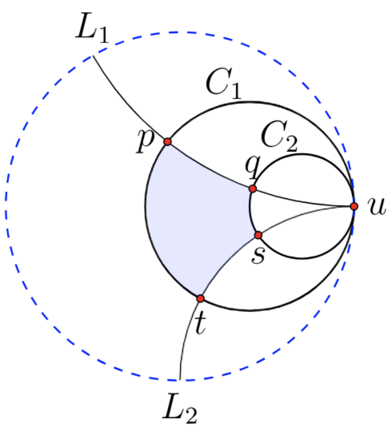
4. Determine the area of the "triangular" region pictured below.



   (a) What is the image of this region under in the disk model of hyperbolic geometry?

   (b) Explain part (a) in light of the theorem for area of a hyperbolic triangle.

5. Consider the four-sided figure $pqst$ in the Poincaré disk model. shown in the following diagram. This figure is determined by two horocycles C_1 and C_2, and two hyperbolic lines L_1 and L_2 all sharing the same ideal point.



   (a) By rotation about the origin if necessary, assume the common ideal point is i and use the figure to the upper half-plane. What does the transferred figure look like in the Upper Half Plane model? Answer parts (b)-(d) by using this transferred version of the figure.

   (b) Prove that the hyperbolic lengths of sides pq and st are equal.

   (c) Let c equal the hyperbolic length of the leg PT along the larger radius horocycle C_1, and let d equal the hyperbolic length of the leg SQ on C_2. Show that $c = e^x d$ where x is the common length found in part (b).

   (d) Prove that the area of the four-sided figure is $c - d$.

6. Below is M.C. Escher's Circle Limit IV and schematic of his work. Somehow he created these without formal training in hyperbolic geometry. Identify three different types of isometries that act on this figure and give the orders of each.