Mon. wk 4

Error in notes, typo in text

Ch. 2

Composing Maps:

$T_1(z) = A\bar{z} + \bar{b}$

$T_2(\bar{z}) = C\bar{z} + d$

$$T_1 \circ T_2(z) = T_1(\underbrace{C\bar{z}+\bar{d}}_{\text{vector}}) = A(C\bar{z}+\bar{d}) + \bar{b} = AC\bar{z} + A\bar{d} + b$$

Exercise: 2.1

Linear Algebra $\Rightarrow$ Any linear map $\alpha: \mathbb{R}^n \to \mathbb{R}^m$ is given by a matrix. Why?

the linear map sends the standard frame $(\bar{e}_1, \bar{e}_2, \dots, \bar{e}_n)$ to columns of A

Idea: T is given, T does something to $\bar{e}_1, \dots, \bar{e}_n$.

$A = [\, T(e_1) \quad T(e_2) \quad \dots \quad T(e_n) \,]$

this "works" b/c.

$\alpha: \mathbb{R}^n \to \mathbb{R}^m \longleftrightarrow A$ $n \times m$ matrix

$\beta: \mathbb{R}^m \to \mathbb{R}^l \longleftrightarrow B$ $m \times l$ matrix

linear maps

gives $B \circ \alpha(\bar{v}) \Leftrightarrow BA(\bar{v})$

Composition of linear maps IS matrix multiplication

matrices

A, B

$\bar{A}_i$ = ith column of A

$\bar{e}_i$ = ith std. basis vector

A^1_i = ith element of 1st column

$$A = [\underbrace{\textstyle\sum \bar{A}^1_i \bar{e}_i}_{\text{columns}} \quad \underline{\textstyle\sum \bar{A}^2_i \bar{e}_i} \quad \underline{\textstyle\sum \bar{A}^3_i \bar{e}_i} \quad \dots \quad \underline{\textstyle\sum \bar{A}^n_i \bar{e}_i}\,]$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \left[\underline{1\begin{pmatrix}1\\0\end{pmatrix} + 3\begin{pmatrix}0\\1\end{pmatrix}} \quad 2\begin{pmatrix}1\\0\end{pmatrix} + 4\begin{pmatrix}0\\1\end{pmatrix}\right]$$

$$\begin{array}{ccc} \bar{e}_i & \xrightarrow{\alpha} & A\bar{e}_i = \\ \Big\downarrow \beta & & \Big\downarrow \beta \\ B\bar{e}_i & & \beta(\alpha(\bar{e}_i)) \end{array}$$

composition

$\beta(\alpha(\bar{e}_i)) = B[e_{i1}\bar{A}_1 + e_{i2}\bar{A}_2 + \dots + e_{im}\bar{A}_m]$

$= e_{i1} \cdot B(\bar{A}_1) + e_{i2} B(\bar{A}_2) + \dots + e_{im} B(\bar{A}_m)$

$= BA(\bar{e}_i)$

matrix mult.

the set of all rigid motions form a group: eg, $T(x) = A\bar{x} + \bar{b}$

1. id : $T(x) = x$, $A = I$, $\bar{b} = \bar{0}$
2. inverse : If $T(\bar{x}) = A\bar{x} + \bar{b}$ what is T^{-1} ?

$$A\bar{x} + \bar{b} = \bar{y}$$
$$Ax = \bar{y} - \bar{b}$$
$$x = \underbrace{A^{-1}}_{\text{exists}}(\bar{y} - \bar{b})$$

exists b/c $A \in O(n)$, $|A| = 1$

3. closed : $T_1 \circ T_2(x) = \underbrace{AC}_{\text{matrix}}(x) + \overbrace{Ad + b}^{\text{a vector}} = Ex + \bar{f}$

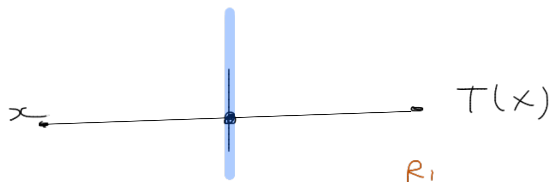
$AC = E \in O(n)$
$\bar{f} \in \mathbb{R}^n$

4. associative : yes. see exercise.

theorem: Every motion of $\mathbb{R}^n$ is generated by reflections.

idea: Let T be a motion. If $T(x) = x$ $\forall$ $x \in \mathbb{R}^n \Rightarrow T = id$
$= R \circ R$
w/ R = some reflection.

Else, $\exists$ some $x \in \mathbb{R}^n$ s.t. $T(x) \neq x$.



Find a reflection (R_1) taking $x \longmapsto T(x)$.
Algorithm: reflect across $\perp$ bisector

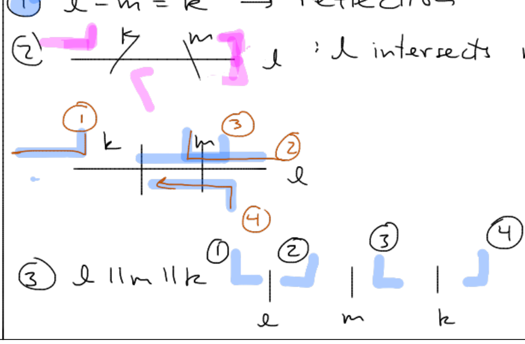
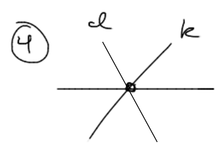
So $R_1 \circ T(x) = x$. If $R_1 \circ T = id$, done b/c $\Rightarrow T = R_1$

Repeat.

this process stops after @ most $n+1$ reflections. Here's why.

$\mathbb{R}^2$

Table of Products of Reflections

Product of two reflections	① $l \parallel m \Rightarrow R_1 \circ R_2 =$ translation ② l intersects m @ angle $\theta \Rightarrow R_1 \circ R_2 = Rot_{2\theta}(P)$
Product of 3 reflections	① $l = m = k \Rightarrow$ reflection ② : l intersects m @ point outside k. ③ $l \parallel m \parallel k$  ④  $\Rightarrow$ reflection (1 fixed point)