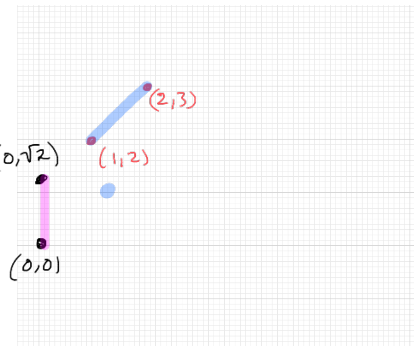
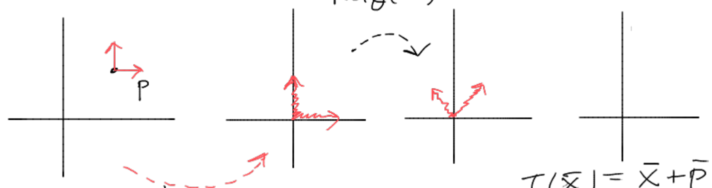
1.11/



Translate, then rotate:

Method for rotating by angle θ about P, in coordinates.

$$Rot_\theta(\bar{0}, \bar{x}) = R(\bar{x}) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$



$T^{-1}(x) = \bar{x} - \bar{p}$

$T(\bar{x}) = \bar{x} + \bar{p}$

$$T \circ R \circ T^{-1}(\bar{x}) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}(\bar{x} - \bar{p}) + \bar{p}$$

composition of motions

rotation about p, by angle theta

In ex. above:

Translate First

id. $(1,2) \longleftrightarrow \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$T_1(\bar{x}) = \bar{x} + \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ gives same map as

$$\boxed{\varphi(\bar{x}) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}(\bar{x}) + \begin{pmatrix} 1 \\ 2 \end{pmatrix}}$$

This relates to (1.8)!

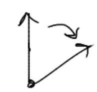
Find angle of Rotation:

equiv. angle b/w $(0, \sqrt{2})$ & $(1,1) = T^{-1}\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$$\theta = \cos^{-1}\left(\frac{(0,\sqrt{2}) \cdot (1,1)}{\sqrt{2} \cdot \sqrt{2}}\right) = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \pi/4$$

we will rotate by $-\pi/4$

talcing



$$\varphi(\bar{x}) = \begin{pmatrix} \cos^{-\pi}/4 & -\sin^{-\pi}/4 \\ \sin^{-\pi}/4 & \cos^{\pi}/4 \end{pmatrix}(\bar{x}) + \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \frac{\sqrt{2}}{2}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}(\bar{x}) + \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

From 1.8 we know

$Ax + b = x$ has some solution.

this $\varphi(x) \overset{\text{also}}{=} Rot_p(\theta)$ Find that point P.

From 1.8 $P =$

1.8/ $A\bar{x} + b = \bar{x}$ $\quad A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

$A\bar{x} - \bar{x} = -\bar{b}$

$A\bar{x} - I\bar{x} = -\bar{b}$

$(A - I)\bar{x} = -\bar{b}$

$$\begin{pmatrix} \cos\theta - 1 & -\sin\theta \\ \sin\theta & \cos\theta - 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -b_1 \\ -b_2 \end{pmatrix}$$

System:

$(\cos\theta - 1)x_1 - \sin\theta\, x_2 = -b_1$

$(\sin\theta)x_1 + (\cos\theta - 1)x_2 = -b_2$

$$x_1 = \frac{-b_2}{\sin\theta} - \frac{(\cos\theta - 1)}{\sin\theta}x_2 = \frac{-b_2 - (\cos\theta - 1)x_2}{\sin\theta}$$

sol'n $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \dfrac{-b_2 - (\cos\theta - 1)x_2}{\sin\theta} \\ x_2 \end{pmatrix} =$

$$(\cos\theta - 1)\left(\frac{-b_2 - (\cos\theta - 1)x_2}{\sin\theta}\right) - \frac{\sin^2\theta\, x_2}{\sin\theta} =$$

$$\frac{-b_2(\cos\theta - 1) - (\cos\theta - 1)^2 x_2 - \sin^2\theta\, x_2}{\sin\theta} = \frac{-b_2(\cos\theta - 1) - \cos^2\theta\, x_2 + 2\cos\theta\, x_2 - x_2 - \sin^2\theta\, x_2}{\sin\theta}$$

$$= \frac{-b_2(\cos - 1) + 2(\cos\theta - 1)x_2}{\sin\theta} = \underbrace{\frac{(\cos\theta - 1)(2x_2 - b_2)}{\sin\theta} = -b_1}_{\text{solve for } x_2}$$

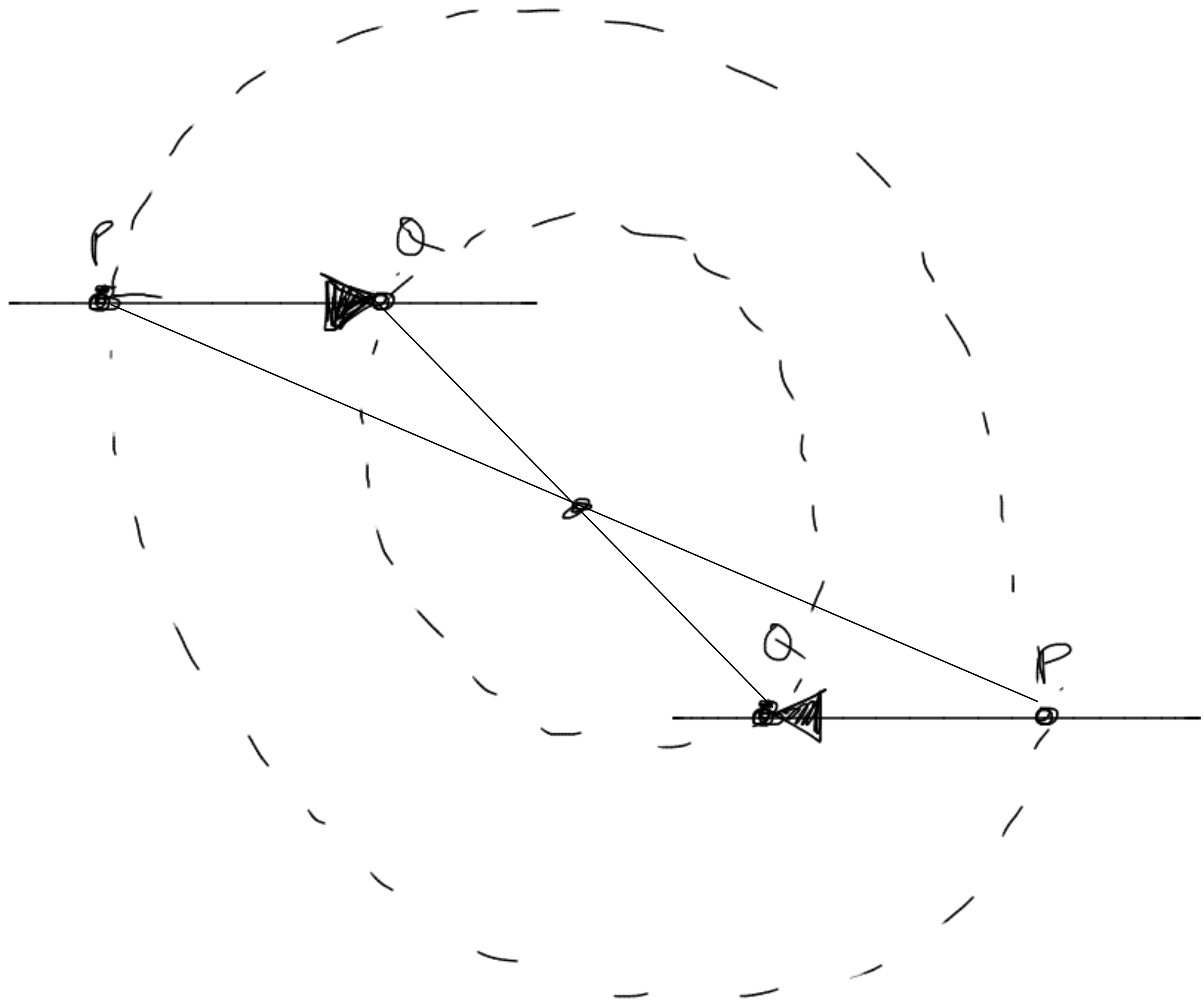
$$2x_2 - b_2 = \frac{-\sin\theta\, b_1}{\cos\theta - 1}$$

$$x_2 = \frac{1}{2}\,\frac{-\sin\theta\, b_1 + b_2(\cos\theta - 1)}{\cos\theta - 1} = \frac{1}{2}\left(b_2 - \left(\frac{\sin\theta}{\cos\theta - 1}\right)b_1\right) = \frac{1}{2}\left(b_2 - \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} \cdot b_1\right)$$

$$\boxed{\begin{array}{l|l} 2\cos^2\frac{\theta}{2} = \cos\theta + 1 & \sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ 2\underbrace{(\cos^2\tfrac{\theta}{2} - 1)} = \cos\theta - 1 & \\ 2\sin^2\frac{\theta}{2} = \cos\theta - 1 & \end{array}} = \frac{1}{2}\left(b_2 - \cot\left(\frac{\theta}{2}\right)b_1\right)$$

1.10

(c)



Composite of two 1/2 turns = translation