

**Exam 1** (each problem is worth 100 points)

#	1	2	3	4	5	6	7	8	Av
Points									

# 1: Find the explicit solution of the initial value problem and state the interval of existence.

$$\frac{dy}{dx} = \frac{x}{y(x^2 - 1)}, \quad y(0) = 1$$

[The following area contains multiple lines of text that have been completely redacted with black ink.]

# 2: Consider the differential equation

$$\frac{dy}{dx} = \frac{xy}{x-1} = f(x,y)$$

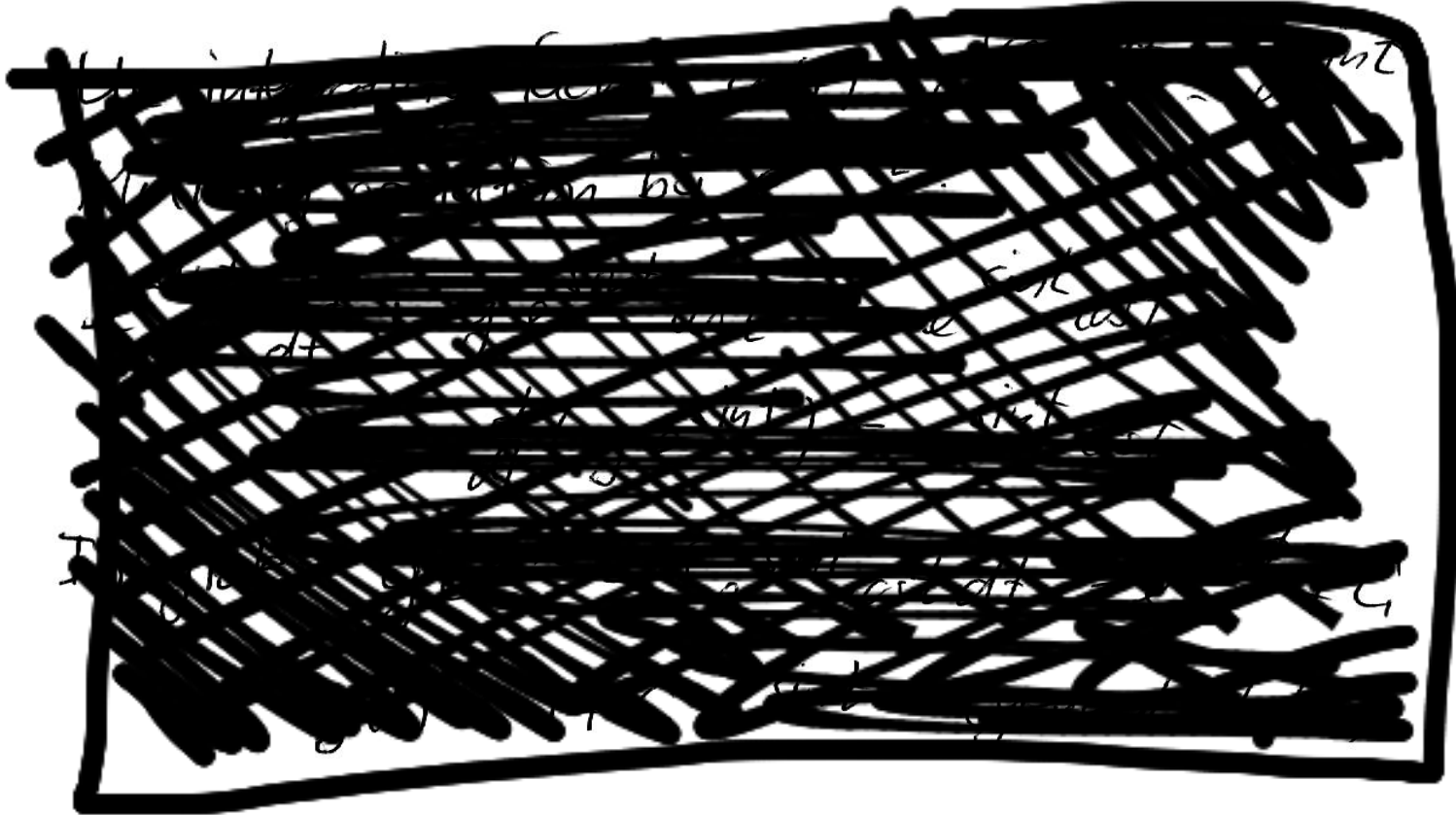
(a) Find the general solution.

(b) Does the equation admit a solution with initial condition  $y(1) = 1$ ? If yes, find the solution. If not, explain why this does not contradict the basic theorem on the existence of solutions (see next page).

*[The following text is heavily obscured by black scribbles and is largely illegible. Some faint words like "y", "x", and "initial condition" are visible through the ink.]*

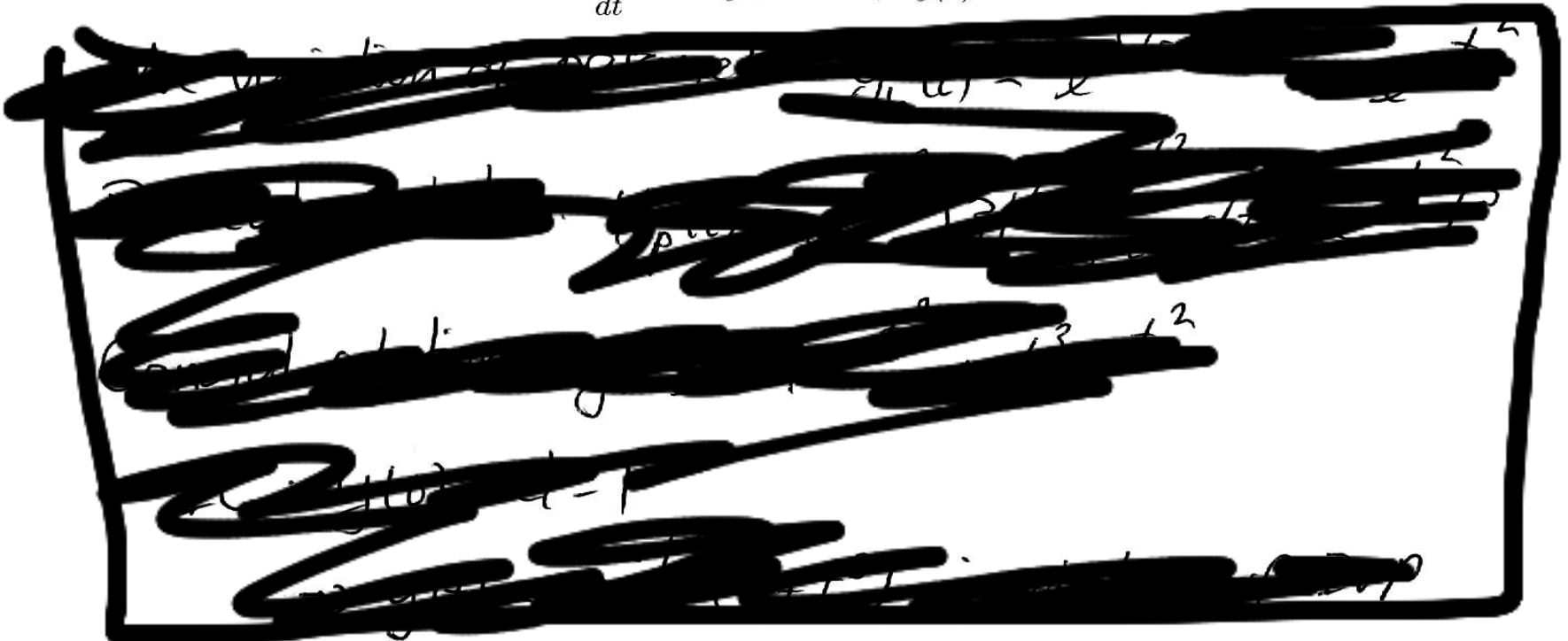
# 3: (a) Find the general solution of the following equation:

$$\frac{dy}{dt} + y \cos t = \cos t$$



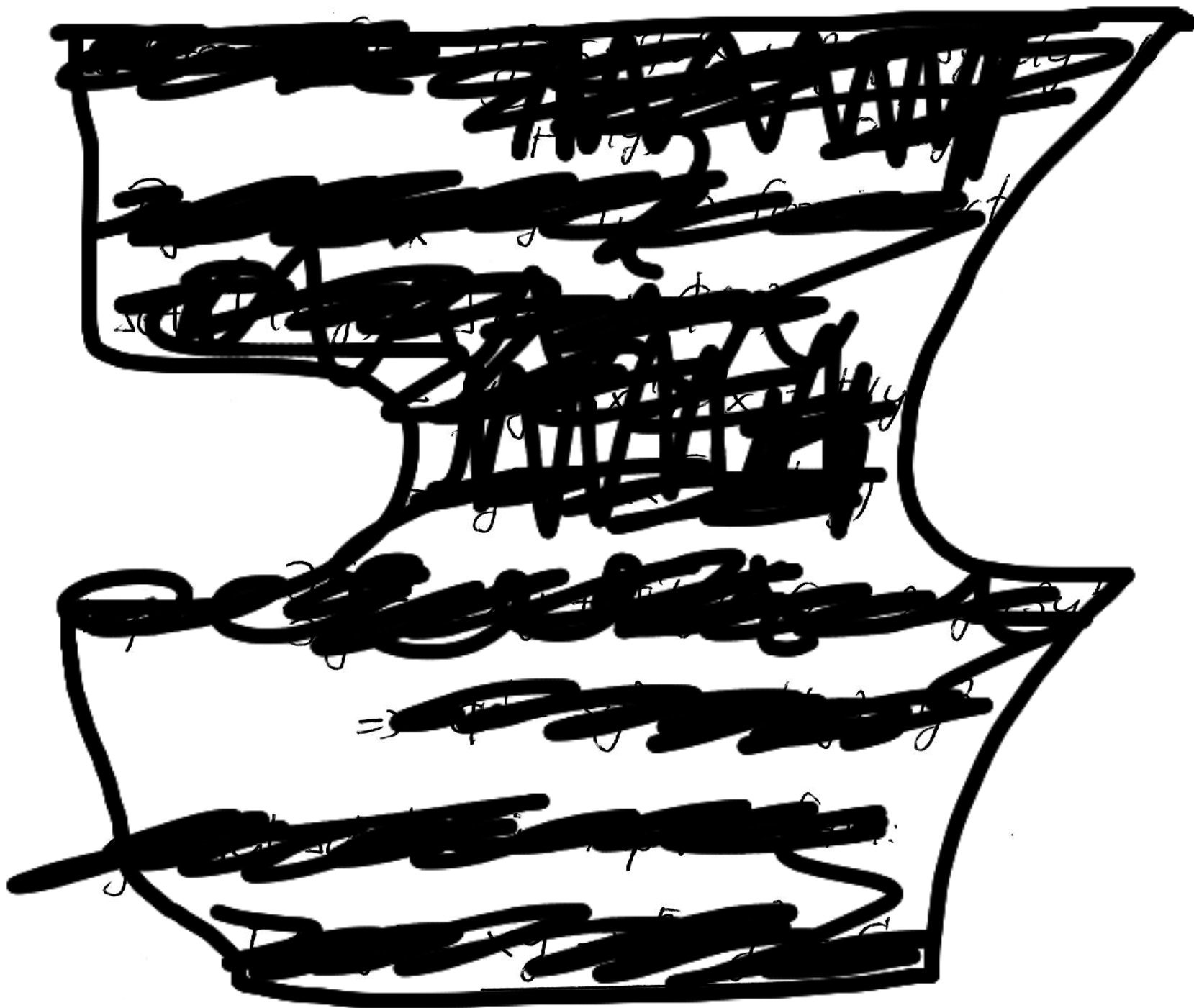
(b) Find the solution of the following initial value problem:

$$\frac{dy}{dt} = -2ty + 3t^2 e^{-t^2}, \quad y(0) = 1$$



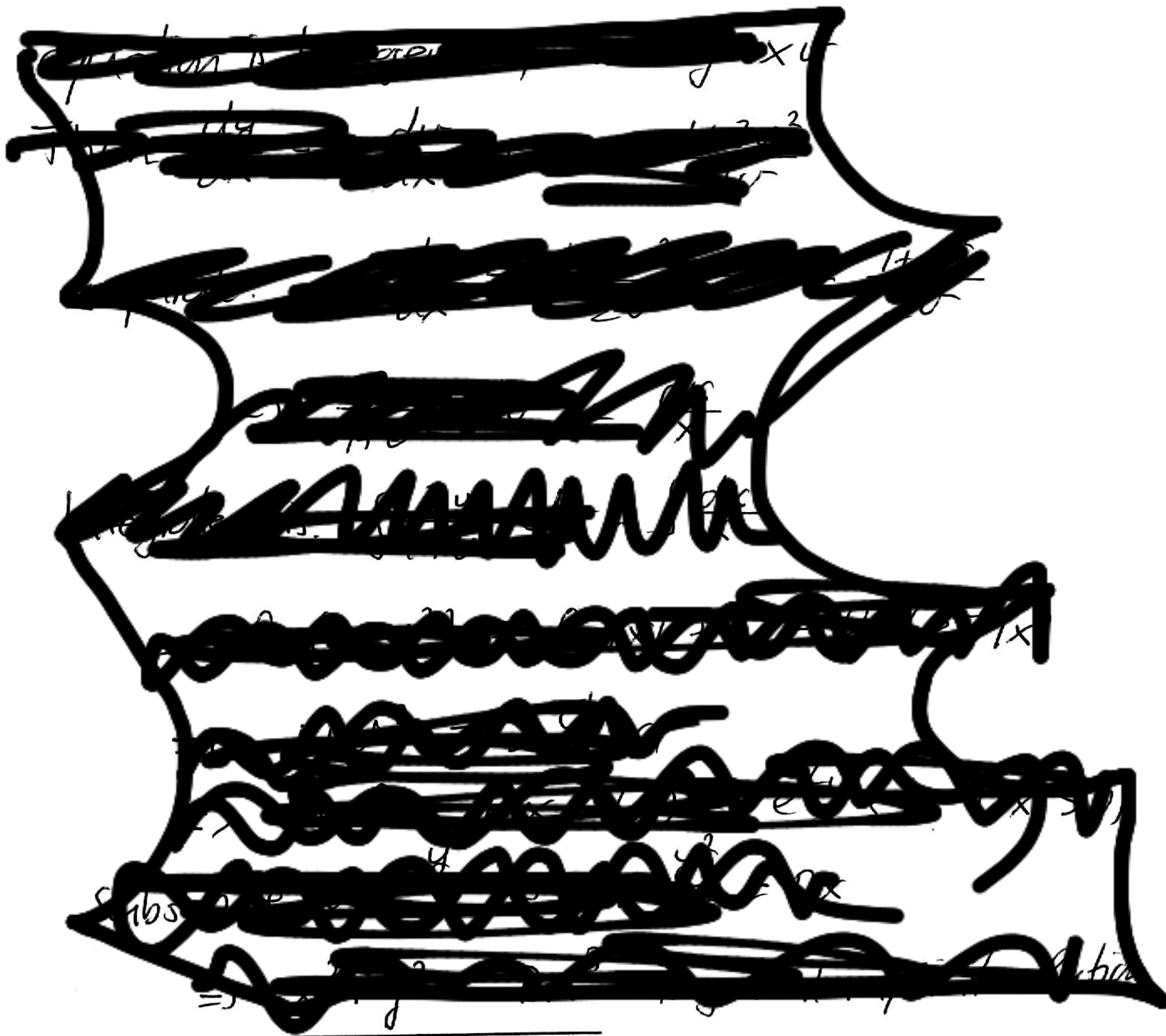
# 4: Find the general solution in implicit form for the equation below. *Hint:* The differential form associated with this equation is exact.

$$\frac{dy}{dx} = \frac{5x^4 - y^2}{2xy + 3y^2}$$



‡ 5: Find the general solution in implicit form for the equation below. *Hint:* The equation is homogeneous.

$$\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}$$



# 6: Newton's law of cooling is given by

$$\frac{dT}{dt} = -k(T - R),$$

where  $T(t)$  is the temperature at time  $t$ ,  $R$  is the surrounding temperature, and  $k$  is a positive constant. Suppose a cold bottle of milk at  $40^\circ$  F is placed into a warm room at  $70^\circ$  F. Suppose 10 minutes later, the temperature of the milk is  $50^\circ$  F. What is the temperature of the milk 20 minutes after the milk was placed into the room? Use the following table for your calculations. Note  $\ln(x) = -\ln(1/x)$ .

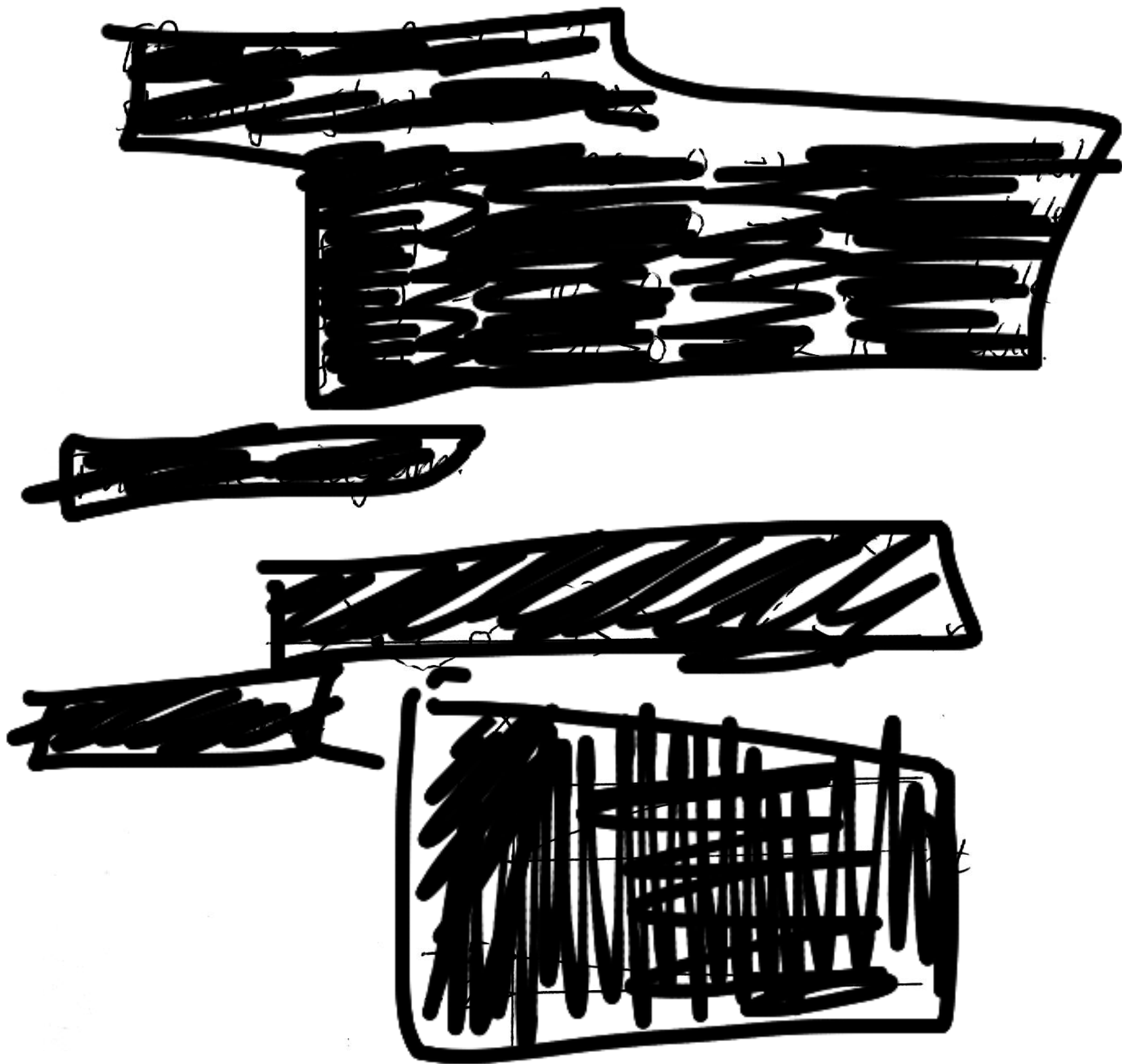
$x$	0.8	1	1.5
$\ln x$	-0.2	0	0.4
$e^{-x}$	0.4	0.35	0.2



# 7: Consider the autonomous equation

$$\frac{dx}{dt} = (x+2)(x+1)(x-1)(x-2) = x^4 - 5x^2 + 4 = f(x)$$

Find all equilibrium points, classify their stability, and sketch the phase line diagram. Use this information to sketch the solution curves in the  $(t, x)$ -plane ( $t \geq 0$ ) for the initial conditions  $x(0) = -2$ ,  $x(0) = -1.5$ ,  $x(0) = 0$ ,  $x(0) = 1$ .



# 8: Define a matrix  $A$ , and vectors  $\mathbf{x}$  and  $\mathbf{b}$ , and express the system of equations below in matrix vector form  $A\mathbf{x} = \mathbf{b}$ . Use the augmented matrix to solve for  $\mathbf{x}$  and express your answer in parametric form if the solution is not unique.

**NOT ON EXAM!**

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &= 5 \\ x_2 - x_3 &= 3 \\ 2x_1 + 4x_2 + 6x_3 + 8x_4 &= 10 \end{aligned}$$

~~$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & -1 & 0 \\ 2 & 4 & 6 & 8 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 5 \\ 3 \\ 10 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$~~

~~augment system~~

~~augmented matrix~~

~~$\left[ \begin{array}{cccc|c} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & -1 & 0 & 3 \\ 2 & 4 & 6 & 8 & 10 \end{array} \right]$~~

~~$\begin{aligned} 1) & R_3 - 2R_1 \\ 2) & R_2 - R_1 \end{aligned}$~~

~~$\left[ \begin{array}{cccc|c} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & -1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$~~

~~Free variables:  $x_3, x_4$       Free variables:  $x_3, x_4$~~

~~Express the solution in parametric form.~~

~~$\begin{aligned} x_1 &= 5 - 2x_2 - 3x_3 - 4x_4 \\ x_2 &= 3 + x_3 \end{aligned}$~~



