Spring 11, MATH 340, Section 220 *Name* ....

Exam 1 (each problem is worth 100 points)

♯	1	2	3	4	5	6	7	8	Av
Points									

♯ 1: Find the explicit solution of the initial value problem and state the interval of existence.

$$\frac{dy}{dx} = \frac{x}{y(x^2-1)}, \quad y(0) = 1$$

[redacted]

♯ 2: Consider the differential equation

$$\frac{dy}{dx} = \frac{xy}{x-1}. = f(x,y)$$

(a) Find the general solution.

(b) Does the equation admit a solution with initial condition $y(1) = 1$? If yes, find the solution. If not, explain why this does not contradict the basic theorem on the existence of solutions (see next page).

[redacted]

♯ 3: (a) Find the general solution of the following equation:

$$\frac{dy}{dt} + y\cos t = \cos t$$

[redacted]

(b) Find the solution of the following initial value problem:

$$\frac{dy}{dt} = -2ty + 3t^2 e^{-t^2}, \quad y(0) = 1$$

[redacted]

♯ 4: Find the general solution in implicit form for the equation below. *Hint:* The differential form associated with this equation is exact.

$$\frac{dy}{dx} = \frac{5x^4 - y^2}{2xy + 3y^2}$$

[redacted]

♯ 5: Find the general solution in implicit form for the equation below. *Hint:* The equation is homogeneous.

$$\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}$$

[redacted]

♯ 6: Newton's law of cooling is given by

$$\frac{dT}{dt} = -k(T - R),$$

where $T(t)$ is the temperature at time t, R is the surrounding temperature, and k is a positive constant. Suppose a cold bottle of milk at 40^o F is placed into a warm room at 70^o F. Suppose 10 minutes later, the temperature of the milk is 50^o F. What is the temperature of the milk 20 minutes after the milk was placed into the room? Use the following table for your calculations. Note $\ln(x) = -\ln(1/x)$.

x	0.8	1	1.5
$\ln x$	-0.2	0	0.4
e^{-x}	0.4	0.35	0.2

[redacted]

♯ 7: Consider the autonomous equation

$$\frac{dx}{dt} = (x+2)(x+1)(x-1)(x-2) = x^4 - 5x^2 + 4. \quad = f(x)$$

Find all equilibrium points, classify their stability, and sketch the phase line diagram. Use this information to sketch the solution curves in the (t, x)-plane ($t \geq 0$) for the initial conditions $x(0) = -2$, $x(0) = -1.5$, $x(0) = 0$, $x(0) = 1$.

[redacted]

♯ 8: Define a matrix A, and vectors $\mathbf{x}$ and $\mathbf{b}$, and express the system of equations below in matrix-vector form $A\mathbf{x} = \mathbf{b}$. Use the augmented matrix to solve for $\mathbf{x}$ and express your answer in parametric form if the solution is not unique.

NOT ON EXAM!

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &= 5 \\ x_2 - x_3 &= 3 \\ 2x_1 + 4x_2 + 6x_3 + 8x_4 &= 10 \end{aligned}$$