Solutions

MA115 Exam 3

1. Make the Unit Circle with the conventional points. You don't need to prove why each point has a certain value, just show me you know the points and angles. Show both degrees and radians.

2. Let θ be an acute angle of a right triangle. Define $\sin\theta$, $\cos\theta$, $\tan\theta$, $\csc\theta$, $\sec\theta$, and $\cot\theta$ in terms of side lengths of the triangle.

3. Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$. Define $\sin\theta$, $\cos\theta$, $\tan\theta$, $\csc\theta$, $\sec\theta$, and $\cot\theta$ in terms of x, y, and r.

4. State the domain and range of $\sin\theta$, $\cos\theta$, $\tan\theta$, $\sin^{-1}\theta$, $\cos^{-1}\theta$, and $\tan^{-1}\theta$.

5. Write $\tan\theta$, $\csc\theta$, $\sec\theta$, and $\cot\theta$ in terms of $\sin\theta$ and/or $\cos\theta$.

6. Evaluate the following exactly:

(a) $\cos(\frac{5\pi}{2})$

(b) $\sin(3645°)$

(c) $\tan(-210°)$

(d) $\sin(\frac{-31\pi}{2})$

(e) $\sec(\frac{4\pi}{3})$

(f) $\tan(\frac{7\pi}{2})$

(g) $\sec(-390°)$

(h) $\csc(450°)$

(i) $\cos(50\pi)$

(j) $\cot(-60°)$

7. Evaluate the following exactly:

(a) $\tan^{-1}(-\sqrt{3})$

(b) $\sin^{-1}(\frac{\sqrt{3}}{2})$

(c) $\cos^{-1}(\frac{1}{2})$

(d) $\sin^{-1}(\frac{-\sqrt{2}}{2})$

(e) $\cos^{-1}(-2)$

(f) $\tan^{-1}(0)$

(g) $\cos^{-1}(\tan(\frac{\pi}{4}))$

(h) $\sin^{-1}(\sin(\frac{11\pi}{6}))$

(i) $\cos^{-1}(\cos(-30°))$

(j) $\tan(\cos^{-1}(\frac{4}{5}))$

(k) $\sec(\sin^{-1}(\frac{5}{13}))$

(l) $\csc(\tan^{-1}(-\frac{3}{4}))$

8. Write an algebraic expression that is equivalent to the given expression for the following:

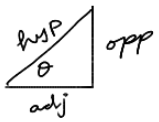
(a) $\cot(\sin^{-1}(\frac{2x}{5}))$

(b) $\cos(\tan^{-1}(x+3))$

9. State the period, asymptotes, x-intercepts, y-intercepts, maximum y-value, and minimum y-value of ONE of the following graphs: $y = \sin x$, $y = \cos x$, $y = \tan x$, $y = \csc x$, $y = \sec x$, or $y = \cot x$.

Bonus: What is the total number of counties in the Upper Peninsula of Michigan?

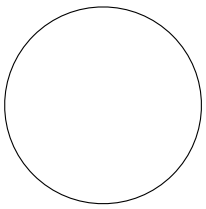
2. Let θ be an acute angle of a right triangle. Define $\sin\theta$, $\cos\theta$, $\tan\theta$, $\csc\theta$, $\sec\theta$, and $\cot\theta$ in terms of side lengths of the triangle.

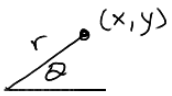


$\sin\theta = \frac{opp}{hyp}$ $\quad \cos\theta = \frac{adj}{hyp}$ $\quad \tan\theta = \frac{opp}{adj}$

$\csc\theta = \frac{hyp}{opp}$ $\quad \sec\theta = \frac{hyp}{adj}$ $\quad \cot\theta = \frac{adj}{opp}$

3. Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$. Define $\sin\theta$, $\cos\theta$, $\tan\theta$, $\csc\theta$, $\sec\theta$, and $\cot\theta$ in terms of x, y, and r.





$x = r\sin\theta$ so $\sin\theta = \frac{x}{r}$

$x = r\cos\theta$ so $\cos\theta = \frac{y}{r}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ so $\tan\theta = \frac{y}{x}$

$\csc\theta = \frac{r}{x}$

$\sec\theta = \frac{r}{y}$

$\cot\theta = \frac{x}{y}$

4. State the domain and range of $\sin\theta$, $\cos\theta$, $\tan\theta$, $\sin^{-1}\theta$, $\cos^{-1}\theta$, and $\tan^{-1}\theta$.

Function	Domain	Range
$\sin\theta$	$(-\infty, \infty)$	$[-1, 1]$
$\cos\theta$	$(-\infty, \infty)$	$[-1, 1]$
$\tan\theta$	$\mathbb{R} - \{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \text{etc}\}$ i.e., $\{x \in \mathbb{R} \mid x \neq \frac{(2k+1)\pi}{2} \text{ for } k \in \mathbb{Z}\}$	
$\sin^{-1}\theta$	$[-1, 1]$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1}\theta$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}\theta$	$(-\infty, \infty)$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

5. Write $\tan\theta$, $\csc\theta$, $\sec\theta$, and $\cot\theta$ in terms of $\sin\theta$ and/or $\cos\theta$.

$\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\csc\theta = \frac{1}{\sin\theta}$, $\sec\theta = \frac{1}{\csc\theta}$, $\cot\theta = \frac{\cos\theta}{\sin\theta}$

6. Evaluate the following exactly:

(a) $\cos(\frac{5\pi}{2}) = 0$ $\quad 5\pi/2 - 2\pi = 5\pi/2 - 4\pi/2 = \pi/2 \quad \cos(\pi/2) = 0$

(b) $\sin(3645°) = \sin(3600 + 45) = \sin(10 \cdot 360 + 45) = \sin 45 = \frac{\sqrt{2}}{2}$

(c) $\tan(-210°) = -\tan(210°) = -\tan(210° - 180°) = -\tan 30 = -\frac{\sqrt{3}}{3}$

(d) $\sin(\frac{-31\pi}{2}) = -\sin(\frac{31\pi}{2}) = -\sin(\frac{32\pi}{2} - \frac{\pi}{2}) = -\sin(16\pi - \pi/2) = -\sin(-\pi/2) = 1$

(e) $\sec(\frac{4\pi}{3}) = \frac{1}{\cos(4\pi/3)} = 1/(-1/2) = -2$

(f) $\tan(\frac{7\pi}{2})$ = undefined

(g) $\sec(-390°) = 1/\cos(-390) = 1/\cos(390) = 1/\cos(360+30) = 1/\cos(30) = 1/\sqrt{3}/2 = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

(h) $\csc(450°) = 1/\sin(450) = 1/\sin(360+90) = 1/\sin(90) = 1$

(i) $\cos(50\pi) = \cos(2\pi(25)) = \cos(2\pi) = 1$

(j) $\cot(-60°) = 1/\tan(-60) = 1/-\tan(60) = 1/-\sqrt{3} = -\frac{\sqrt{3}}{3}$

7. Evaluate the following exactly:

(a) $\tan^{-1}(-\sqrt{3}) = \theta$ w/ $\tan(\theta) = -\sqrt{3}$, $\theta = -\frac{\pi}{3}$

(b) $\sin^{-1}(\frac{\sqrt{3}}{2}) = \pi/3$

(c) $\cos^{-1}(\frac{1}{2}) = \pi/3$

(d) $\sin^{-1}(\frac{-\sqrt{2}}{2}) = -\pi/4$

(e) $\cos^{-1}(-2)$ DNE

(f) $\tan^{-1}(0) = 0$

(g) $\cos^{-1}(\tan(\frac{\pi}{4})) = \cos^{-1}(1) = 0$

(h) $\sin^{-1}(\sin(\frac{11\pi}{6})) = \sin^{-1}(\sin(-\pi/6)) = -\pi/6$

(i) $\cos^{-1}(\cos(-30°)) = \cos^{-1}(\cos(30)) = 30$

(j) $\tan(\cos^{-1}(\frac{4}{5}))$  $\Rightarrow = 3/4$

(k) $\sec(\sin^{-1}(\frac{5}{13}))$  $\Rightarrow 13/12$

(l) $\csc(\tan^{-1}(-\frac{3}{4}))$  $\Rightarrow \frac{1}{(-4/5)} = -5/4$ or use even/odd properties

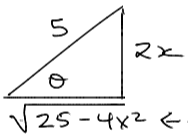
$\csc(\tan^{-1}(-\frac{3}{4})) = \csc(-\tan^{-1}(\frac{3}{4}))$

$= -\csc(\tan(\frac{3}{4}))$

$= -5/4$

8. Write an algebraic expression that is equivalent to the given expression for the following:

$\theta = \sin^{-1}(\frac{2x}{5}) \rightarrow \sin\theta = \frac{2x}{5}$ so



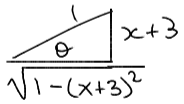
$\sqrt{25-4x^2}$ ← pythag. thm

(a) $\cot(\sin^{-1}(\frac{2x}{5}))$ → $\cot(\theta) = \frac{\sqrt{25-4x^2}}{2x}$

(b) $\cos(\tan^{-1}(x+3))$

① $\theta = \tan^{-1}(x+3)$

② $\tan\theta = x+3 = \frac{x+3}{1}$

③ 

④ $\cos\theta = \sqrt{1-(x+3)^2}$

9. State the period, asymptotes, x-intercepts, y-intercepts, maximum y-value, and minimum y-value of ONE of the following graphs: $y = \sin x$, $y = \cos x$, $y = \tan x$, $y = \csc x$, $y = \sec x$, or $y = \cot x$.

Function	period	asymptotes	x-int	y-int	max y	min y
$\sin(x)$	2π	none	$k\pi$	0	1	-1
$\cos(x)$	2π	none	$(2k+1)\frac{\pi}{2}$	1	1	-1
$\tan(x)$	π	$(2k+1)\frac{\pi}{2}$	$k\pi$	0	none	none
$\csc(x)$	2π	$k\pi$	none	0	none	none
$\sec(x)$	2π	$(2k+1)\frac{\pi}{2}$	none	1	none	none
$\cot(x)$	π	$k\pi$	$(2k+1)\frac{\pi}{2}$	none	none	none