MA 115 Exam 4 Name:

1. Prove the following double angle formulas using the angle sum formulas.

   a) $\sin(2x) = 2\sin(x)\cos(x)$

   $\sin(2x) = \sin(x+x)$
   $= \sin(x)\cos(x) + \cos(x)\sin(x)$
   $= \sin(x)\cos(x) + \sin(x)\cos(x)$
   $= 2\sin(x)\cos(x)$

   b) $\cos(2x) = \cos^2(x) - \sin^2(x)$

   $\cos(2x) = \cos(x+x)$
   $= \cos(x)\cos(x) - \sin(x)\sin(x)$
   $= \cos^2(x) - \sin^2(x)$

2. Find the Exact Values (By hand)(Not showing work is worth 0 pts)

   a) $\sin(\frac{5\pi}{12})$

   $= \sin(\frac{\pi}{4} + \frac{\pi}{6})$
   $= \sin(\frac{\pi}{4})\cos(\frac{\pi}{6}) + \cos(\frac{\pi}{4})\sin(\frac{\pi}{6})$
   $= (\frac{\sqrt{2}}{2})(\frac{\sqrt{3}}{2}) + (\frac{\sqrt{2}}{2})(\frac{1}{2})$
   $= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$
   $= \frac{\sqrt{6}+\sqrt{2}}{4}$

   b) $\cos(\frac{5\pi}{12})$

   $= \cos(\frac{\pi}{4} + \frac{\pi}{6})$
   $= \cos(\frac{\pi}{4})\cos(\frac{\pi}{6}) - \sin(\frac{\pi}{4})\sin(\frac{\pi}{6})$
   $= (\frac{\sqrt{2}}{2})(\frac{\sqrt{3}}{2}) - (\frac{\sqrt{2}}{2})(\frac{1}{2})$
   $= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$
   $= \frac{\sqrt{6}-\sqrt{2}}{4}$

   c) $\tan(\frac{5\pi}{12})$

   $= \tan(\frac{\pi}{4} + \frac{\pi}{6})$
   $= \frac{\tan(\frac{\pi}{4})+\tan(\frac{\pi}{6})}{1-\tan(\frac{\pi}{4})\tan(\frac{\pi}{6})}$
   $= \frac{1+\frac{1}{\sqrt{3}}}{1-(1)(\frac{1}{\sqrt{3}})}$
   $= \frac{\sqrt{3}+1}{\sqrt{3}} * \frac{\sqrt{3}}{\sqrt{3}-1}$
   $= \frac{\sqrt{3}+1}{\sqrt{3}-1}$

3. Find All Solutions to the Following:

a) $2\cos(3x) + 1 = 0$

$u = 3x$

$2\cos(u) + 1 = 0$

$2\cos u = -1$

$\cos u = \frac{-1}{2}$

$u = \frac{2\pi}{3} + 2\pi n, \frac{4\pi}{3} + 2\pi n$

$x = \frac{2\pi}{9} + \frac{2\pi n}{3}, \frac{4\pi}{9} + \frac{2\pi n}{3}$

b) $\left(\tan^2(x) - 1\right)\sin(x) = 0$

$\tan^2(x) - 1 = 0$

$\tan^2 x = 1$

$\tan x = \pm 1$

$x = \frac{\pi}{4} + \frac{\pi}{2}n$

$\sin x = 0$

$x = \pi n$

$x = \frac{\pi}{4} + \frac{\pi}{2}n, \pi n$

c) $\sec(x)\sin(x) - \sec(x) = 0$

$(\sec x)(\sin x - 1) = 0$

$\sec x = 0$ DNE

$\sin x - 1 = 0$

$\sin x = 1$

$x = \frac{\pi}{2} + 2\pi n$

4. Solve for any possible triangles, round to two decimal places if needed. If not possible, indicate so:

a) $a = 2, b = 5, c = 11$

Since $c > a + b$ this is not a triangle

b) $A = 50°, a = 15, B = 60°$

$\frac{15}{\sin 50°} = \frac{b}{\sin 60°} = \frac{c}{\sin 70°}$

$b = \frac{15 \sin 60°}{\sin 50°}, c = \frac{15 \sin 70°}{\sin 50°}$

$a = 15, b \approx 16.96, c \approx 18.40$

$A = 50°, B = 60°, C = 70°$

c) $A = 100°, a = 25, b = 10$

$\frac{a}{\sin 100°} = \frac{10}{\sin B} = \frac{c}{\sin C}$

$B = \sin^{-1}(\frac{10 \sin 100°}{25}) \approx 23.2°, c = \frac{25 \sin 56.8°}{\sin 100°}$

$a = 25, b = 10, c \approx 21.24$

$A = 100°, B \approx 23.2°, C \approx 56.8°$

5. Word Problems

a) A pilot measures the angle of depression to two ships in the water in front of the plane as 30 degrees and 50 degrees respectively. If the pilot is flying at an altitude of 45,000 feet. Draw a picture to represent the problem and determine the distance between the two ships.

Find leg of smaller triangle $l = \frac{45000 \sin 40^\circ}{\sin 50^\circ}$

Note angle opposite the leg we want to find is $90^\circ - 50^\circ = 40^\circ$

$l \approx 37759 ft$

Find leg of Larger triangle $L = \frac{45000 \sin 60^\circ}{\sin 30^\circ}$

Note angle opposite the leg we want to find is $40^\circ + 20^\circ = 60^\circ$

$L \approx 77942$ feet

$x = 77942 - 37759 = 40183$feet

Check for appropriate drawing

b)Points P and Q are separated by a lake. To find the distance between them, a surveyor locates point R on land such that angle PRQ is 130 degrees, the distance from P to R is 250 feet, and the distance from Q to R is 475 feet. Draw a picture to represent the problem and find the distance between points P and Q. Round to the nearest foot

$r^2 = 250^2 + 475^2 - 2(250)(475)\cos 130^\circ$

$c = \sqrt{250^2 + 475^2 - 2(250)(475)\cos 130^\circ}$

$x \approx 664$ feet

Check for appropriate drawing

Extra Credit: Earlier you proved $\cos(2x) = \cos^2(x) - \sin^2(x)$ rewrite $\cos(2x)$ in one or two (for more extra credit!) different forms: