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**Name:**

Math 163      Take-Home Exam 4

Date:            April 16, 2026

Total Points: 70

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## Numerical

Numerical approximations are allowed. Solutions may be on paper or in Desmos, python, R, etc. (You may use  $f'(x)$  notation in Desmos.) Print or email any work you do in Desmos, etc.

1. In the following consider the polar curve  $r = 3 \cos(6\theta) + 1, 0 < \theta < \pi$ .
  - (1.1) Sketch a graph of the polar curve.
  - (1.2) Find the arc length of the curve.
  - (1.3) Using a slider, create parametric functions that convert from polar coordinates to rectangular coordinates.
  - (1.4) Use the slider and these rectangular coordinates to plot a point on the curve whose position depends on the parameter  $c$ .
  - (1.5) Create a function that computes the slope of the tangent line to the curve at the point corresponding to  $c$ . (You'll have to use the product rule to compute  $\frac{dy}{dx}$ .)
  - (1.6) Plot this tangent line on your graph.
  - (1.7) Update you arc length calculation to be in terms of  $c$ . Find the value of  $c$  that gives a length of exactly  $2\pi$ . Plot the point corresponding to this value of  $c$  on your graph.
  - (1.8) Plot on top of your graph the circle with radius 1 and center at the origin.
  - (1.9) What is interesting about these last two steps?
- 2.(2.1) Find the area bound by all loops of the polar curve
$$r = 6 \cos(\theta) + 6 \cos(3\theta) + 6 \cos(5\theta) - 6 \cos(7\theta) - 6 \cos(9\theta)$$
  - (2.2) Plot the circle whose area differs from the area above by at most  $10^{-2}$ .
3. Find the surface area of the torus obtained by rotating the circle  $x^2 + (y - 14)^2 = 49$  about the x-axis.
4. Find the two inflection points (remember where  $\frac{dy}{dx} = 0$ ) of the limaçon  $r = 4 \cos(\theta) - 6$ .

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## Exact

Solutions are to be exact, and completely on paper. No numerical approximations.

5. Find the area of region in the figure  $r = 8 \sin(5\theta)$

6. Find the surface area of the solid obtained by revolving the curve

$$y = \sin(3t), x = \cos(3t)$$

about the x-axis.

7. Solve the following initial value problem:

$$\frac{dy}{dx} = \frac{\sin x}{\cos y}, y(0) = \pi$$

8. Solve the following initial value problem:

$$\frac{dy}{dx} = \frac{(y^2 + 1) \sec^2(x)}{y}, y(0) = 0$$

9. Solve the following initial value problem:

$$y' - 4y = 8x, y(0) = 5$$

10. Solve the following initial value problem:

$$y' - \frac{3}{x}y = 4, y(1) = 7$$