ma516-hw2

Complete the following:

1. Let D be the set of dogs in Marquette, MI at this instant. Let $a \sim b$ if dog a has the same mother as dog b. Show that $\sim$ is an equivalence relation on D, and describe the equivalence classes.
2. Prove that a topology $\mathcal{T}$ on a set X is the discrete topology if and only if $\{x\} \in \mathcal{T}$ for all $x \in X$.
3. Define a topology on $\mathbb{R}$ (by listing the open sets within it) that contains the open sets (0,2) and (1,3) and that contains as few open sets as possible.
4. For each $n \in \mathbb{Z}$, define

$$B(n) = \begin{cases} n & \text{if } n \text{ is odd} \\ n-1, n, n+1 & \text{if } n \text{ is even} \end{cases}$$

Show that the collection $\mathcal{B} = \{B(n) : n \in \mathbb{Z}\}$ is a basis for a topology on $\mathbb{Z}$. Describe the open sets in this topology. The topology is called the *digital line topology.*

5. Let S denote the set of points

$$\{(0, \frac{1}{n}) \in \mathbb{R}^2 : n \in \mathbb{Z}^+\}.$$

(Note that S is a set of points in $\mathbb{R}^2$, not open intervals.) Either prove that $\mathbb{R}^2 - S$ is an open set in the standard topology on $\mathbb{R}^2$ or prove that it is not an open set.

Repeat the question for the set $T = S \cup \{(0, 0)\}$.