# Review & Final Exam Preparation

## Defintions

1. Connected sum of surfaces
2. Orientable vs non-orientable surfaces
3. Genus of a orientable closed surface
4. Retract
5. Deformation retract
6. path
7. homotopy
8. homotopy equivalence
9. fundamental group
10. simple closed curve
11. essential simple closed curve
12. non-separating simple closed curve
13. separating simple closed curve
14. homotopy class of a simple closed curve
15. $X$ is connected if …
16. $X$ is path connected if …
17. $X$ is simply connected if …
18. $X$ is homotopy equivalent to $Y$ if …
19. $X$ is homeomorphic to $Y$ if …
20. $X$ is homotopy equivalent to a point if …
21. $X$ is contractible if …
22. $X$ is compact if …
23. $X$ is a n-manifold if …
24. $X$ is a closed n-manifold if …
25. $X$ is a surface if …

## Theorems

1. State the Brouwer Fixed Point Theorem.

2. State the classification of surfaces.
3. Prove that homeomorphic spaces are homotopy equivalent.
4. Prove that homotopy equivalent spaces have isomorphic fundamental groups.
5. State a theoerm that relates deformation retracts and homotopy equivalences.

## Examples

1. Give an example of a non-orientable surface.
2. Give an example of a non-separating simple closed curve on a torus.
3. Give an example of a separating simple closed curve on a torus.
4. Give an example of a compact subset of $\mathbb{R}^2$.
5. Give an example of a non-compact subset of $\mathbb{R}^2$.
6. The topologist's sine curve is an example of …

## Computations

1. Compute $\pi_1(T^2)$ (the torus)
2. Compute $\pi_1(S^1)$
3. Compute $\pi_1(\mathbb{R}P^2)$
4. Connected sums
   - $\mathbb{R}P^2 \# \mathbb{R}P^2$
   - $\mathbb{R}P^2 \# \mathbb{R}P^2 \# \mathbb{R}P^2$
   - $\mathbb{R}P^2 \# T^2$.
5. Compute a fundamental group using S.V.K.
6. Compute a fundamental group using deformation retracts.
7. Compute $\pi_1$ of the punctured torus.
8. Compute $\pi_1$ of the thrice (3 times) punctured sphere (also draw picture and recognize it as a surface with boundary).
9. Realize the Mobius band as a punctured projective plane.

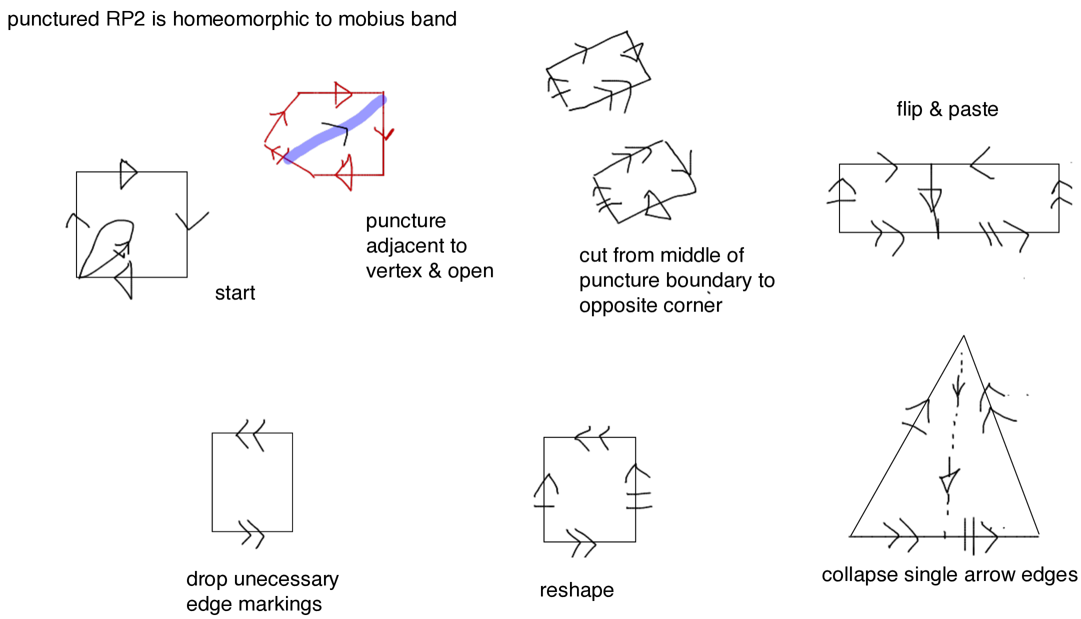


Figure 1: Punctured RP2