Local Neighborhood of a Quiz

1. State the definitions of
    1. a continuous function between topological spaces X and Y.
    2. a closed set in a topological space X.
    3. limit point of a set A in a topological space X.
    4. (local formulation of openness) a set U is open if and only if … (fill in the blank)
    5. Hausdorff space.
    6. a topological space being *discrete*.
    7. a basis for a topology on a set X.

2. True or False Section
    1. True or False: Any intersection of open sets is open.
    2. True or False: Any intersection of closed sets is closed.
    3. True or False: Any union of closed sets is closed.
    4. True or False: Any union of open sets is open.
    5. True or False: A set A is closed if and only if its complement $X \quad A$ is open.
    6. True or False: A set A is open if and only if its complement $X \quad A$ is closed.
    7. True or False: A set A is closed if and only if it contains all of its limit points.
    8. True or False: A set A is open if and only if it contains all of its limit points.

3. Give an example of
    1. a continuous function $f : X \to Y$ between topological spaces X and Y.
    2. a limit point of a set A in a topological space X.
    3. a topological space that is Hausdorff.

1. Short answer questions

    1. Let X be a topological space and let $A \subseteq X$. Prove that $\overline{A}$ is closed.

    2. Find the closure of the graph of the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \sin(1/x)$ for $x \neq 0$ and $f(0) = 0$. (Use Desmos if you like!)

    3. Show the lower limit topology contains proper subsets that are both open and closed.

    4. Do continuous functions preserve limit points? That is, if $f : X \to Y$ is continuous and x is a limit point of a set $A \subseteq X$, is $f(x)$ a limit point of $f(A)$? Prove or give a counterexample.

    5. Let X be a topological space and let $A \subseteq X$. Prove that $\overline{A} = A \cup A'$, where A' is the set of limit points of A.

    6. Let X be a topological space and let $A \subseteq X$. Prove that $\overline{A}$ is the smallest closed set containing A. That is, if C is a closed set such that $A \subseteq C$, then $\overline{A} \subseteq C$.

    7. Is $\mathbb{Q}$ a closed set in $\mathbb{R}$? Prove your answer.