

Midterm Exam
Math 516 - Winter 2026

Name _____

1. Familiar Questions (10 points each)

- (a) Let X be a topological space, and let $A \subset X$ be a subset. Show that $\overline{A} = A \cup A'$, where A' is the set of all limit points of A .
- (b) Consider the lower limit topology τ on \mathbb{R} , which is generated by the basis consisting of intervals $[a, b)$ with $a, b \in \mathbb{R}$. Find a proper subset that is both open and closed in (\mathbb{R}, τ) .
- (c) Give an example of a continuous map that is not an open map. (A sketch is ok, but be sure to explain why your map is continuous and why it is not an open map.)
- (d) Use the local formulation of openness to show that unit circle $S^1 = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$ is closed in \mathbb{R}^2 .
- (e) Describe how the circle and torus are quotient space. (A sketch is ok, but be sure to explain the equivalence relation you are using and why the resulting quotient space is homeomorphic to the circle or torus.)
- (f) Show the finite complement topology on \mathbb{R} is not Hausdorff.
- (g) Show that if $f : X \rightarrow Y$ is a continuous map then $f(\overline{A}) \subset \overline{f(A)}$ for any subset $A \subset X$.

2. New Questions (Choose three, 10 points each)

- (a) Let τ denote the topology generated by the basis consisting of intervals $[a, b)$ with **rational** endpoints, i.e., $a, b \in \mathbb{Q}$. Determine the closure of the set $(\sqrt{5}, \sqrt{6})$.
- (b) Let $f : X \rightarrow Y$ and $g : X \rightarrow Y$ be two continuous maps. Suppose Y is a Hausdorff space, and that there is a dense subset $D \subset X$ such that $f(x) = g(x)$ for all $x \in D$. Show that $f(x) = g(x)$ for all $x \in X$.
- (c) A space X is said to be *homogeneous* if, for every two points $x_1, x_2 \in X$, there is a self-homeomorphism $f : X \rightarrow X$ such that $f(x_1) = x_2$.
 - (a) Prove that homogeneity is a topological property. That is to say, if X is homeomorphic to Y , and X is homogeneous, then Y is also homogeneous.
 - (b) Give an example of a topological space that is homogeneous.
- (c) Recall the definition of boundary of a set A in a topological space X : $\partial A = \overline{A} \cap \overline{X - A}$. Show that ∂A is empty if and only if A is both open and closed.
- (d) Can a set be equal to its boundary? If yes, give an example. If not, explain why not.