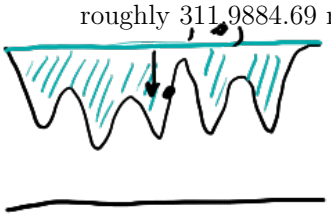
Name:

Sections 1.1 and 1.2 Exercises :: Real Numbers - Exponents & Radicals :: Math 115

1. Verify the tale that the volume of Lake Superior is great enough to cover the continental US in 5 feet of water.

   The average depth of Lake Superior is 483 feet, and its surface area is 31,700 square miles. Convert 31,700 square miles to square feet by multiplying by $\frac{5280^2 ft^2}{1 mi^2}$ and compute the estimated volume of Lake Superior in cubic feet. (The surface area of the continental US is roughly 311,9884.69 miles^2. You'll have to convert this to square feet too.)



Avg Depth

Vol = Avg Depth × Surface Area

LAKE $V = 483\,ft \times 31{,}700\,mi^2 \times \frac{5280^2\,ft^2}{1\,mi^2} = 4.3 \times 10^{14}\,ft^3$

U.S. $4.3 \times 10^{14}\,ft^3 = D \times 3{,}119{,}885\,mi^2 \times \frac{5280^2\,ft^2}{1\,mi^2} = D \cdot 8.6 \times 10^{13}\,ft^2$

$D = \frac{4.3 \times 10^{14}}{8.6 \times 10^{13}} = \frac{1 \times 10^{14}}{2 \times 10^{13}} = \frac{1}{2} \cdot 10 = 5$

2. Can you see Pictured Rocks from atop Mount Marquette?

From the shore

Due to the curvature of the earth the maximum distance D that you can see from the top of a tall building/mountain of height h is estimated by the forumla

$h = \frac{6}{5280} \Rightarrow D \approx 3$

$$D = \sqrt{2rh + h^2}$$

$h = 1{,}200'$ this is the height of Mt. MQT.

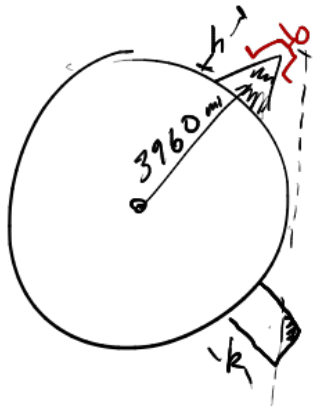
where $r = 3960$mi is the radius of the earth and D and h are also measured in miles. If an object on ~~the~~ horizon is k feet tall, then the maximum distance from which a person can still see it is

$$D = \sqrt{2rh + h^2} + \sqrt{k} * 1.32.$$

$k = 200'$

(Pictured Rocks lakeshore is roughly 45 miles away from Marquette and 200 feet above lake level. Go to the top of Mount Marquette on a clear day and look eastward/southeastward, and confirm your answer.

$$D = \sqrt{2(3960)\left(\frac{1200}{5280}\right) + \left(\frac{1200}{5280}\right)^2} + \sqrt{\frac{200}{5280}} \cdot 1.32 = 42.6$$



3. Set notation helps us communicate collections of numbers effectively. Find the indicated sets if

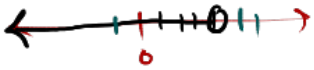
$$A = \{1, 2, 3, 4, 5, 6, 7\}, B = \{2, 4, 6, 8\} \text{ and } C = \{7, 9, 10\}$$

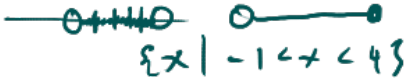
(a) $A \cup B$ = $\{1,2,3,4,5,6,7,8\}$
(b) $A \cap B$ = $\{2,4,6\}$
(c) $B \cap C$ → $= \phi$
(d) $A \cap B \cap C$ = ϕ

4. Find the indicated sets if the set of real #'s x where $x < 4$.

$$A = \{x \mid x < 4\} \text{ and } C = \{x \mid -1 < x \leq 6\}$$

(a) $B \cup C$
(b) $B \cap C$ →





$\{x \mid -1 < x < 4\}$

5. The number line:

Graph the set $(-2, 0) \cup (-1, 1)$
Graph the set $[-4, 6) \cap [0, 8)$

6. Using only the figure in Ex. 80 from Section 1.1 - between what two whole numbers is $\sqrt{2}$?

this has length $\sqrt{2}$ so its between 1 & 2.

7. Name the property illustrated:

$2x + 5 = 5 + 2x$ commutative

$(2x + 5) + 7y = 2x + (5 + 7y)$ associative

$A(C + D) = AC + AD$ distributive

8. Is subtraction commutative? Is the operation of putting your shoes and socks on commutative? → NO

$5 - 4 \neq 4 - 5$

9. Is this true of false? $(A + B)(C + D) = (A + B)C + (A + B)D$

True it just this property

10. Simplify into one power of two. $4^x \cdot 2^y =$ _____ $(2^{2x}) \cdot 2^y = 2^{2x+y}$

11. Simplify $(2^2)^5$ $= 2^{10} = 1024$

12. Simplify $(y+x)^3(y+x)^{-5}$ $= (y+x)^{-2}$

13. Simplify $\left(\frac{xy}{4}\right)^3 \cdot \left(\frac{2x^2}{4y}\right)^5$ $\frac{x^3y^3}{4^3} \cdot \frac{2^5x^{10}}{4^5y^5} = \frac{x^{13}}{2^{11}y^2}$

14. Simplify $(2x^3b^2)(3xb^4)^3$ $= 2x^3b^2 27x^3b^{12} = 54x^6b^{14}$

15. Eliminate negative exponents and simplify: $\frac{3xy^{-2}}{3x^{-3}y^4}$ $= \frac{x \cdot x^4}{y^4 y^2} = \frac{x^5}{y^6}$

16. Simplify $\sqrt[3]{x^4} + \sqrt[3]{x}$ $\sqrt[3]{x^3 \cdot x}$ $x\sqrt[3]{x} + \sqrt[3]{x} = \sqrt[3]{x}(x+1)$

17. Simplify $\sqrt[5]{\frac{32}{x^6}}$ $= \frac{\sqrt[5]{32}}{\sqrt[5]{x^5 \cdot x}} = \frac{2}{x\sqrt[5]{x}}$ $x^2 + 2xy + y^2$

18. Is it ever true that $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$? only true when $a=0$ or $b=0$

19. Simplify $x^{1/2}x^{7/2}$

20. Simplify $(4a^3b^4)^{1/2}$ $= 4^{1/2} \cdot a^{3/2} b^{4/2} = 2a^{3/2}b^2$

21. Rationalize the denominator $\frac{1}{\sqrt{10}}$, $\cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10}}{10}$ $\frac{1}{2}$ $\frac{1}{2}$

22. Rationalize the denominator $\frac{2}{\sqrt{a}}$ $\cdot \frac{\sqrt{a}}{\sqrt{a}} = \frac{2\sqrt{a}}{a}$

23. Rationalize the denominator $\frac{a}{\sqrt[3]{b^2}}$ $= \frac{a}{b^{2/3}} \cdot \frac{b^{1/3}}{b^{1/3}} = \frac{a\sqrt[3]{b}}{b}$

24. Simplify $\sqrt[3]{\frac{-1}{x^6}}$ $= \frac{-1}{x^2}$

$\sqrt[3]{x^6} = x^{6/3} = x^2$

25. Simplify $\left(\frac{x^{-1}yz^{-2}}{y^{-5}zx^{-8}}\right) =$ $x^7 y^6 z^{-3}$

26. Simplify $\sqrt{a^2b^{-1}}\sqrt[4]{a^{-2}b^{-1}}$ $= (a^2b^{-1})^{\frac{1}{2}}(a^{-2}b^{-1})^{\frac{1}{4}}$

$= ab^{-\frac{1}{2}}a^{-\frac{1}{2}}b^{-1/4} = a^{1/2}b^{-3/4}$

27. Simplify by hand $\frac{4.2 \times 10^5}{8.4 \times 10^8}$ $= \frac{1}{2 \times 10^3} = \frac{1}{2000}$

28. As of yesterday, the total number of websites on the internet is 1,175,851,451. Write this number in scientific notation.

1.175851451×10^9

29. A sheet of paper is about .005 inches thick. Write this in scientific notation. How many times can you fold in half a sheet of paper and how thick is it then?

5×10^{-3}