Exam 3 Study Guide :: Math 115 :: Winter 2015

1. **Exponential Functions**
   How long will it take for an investment of \$1000 to double in value if the interest rate is 6.5%, compounded quarterly?

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$2000 = 1000\left(1 + \frac{.065}{4}\right)^{4t}$

$2 = (1.0163)^{4t}$

$\log_2 2 = 4t \log_2(1.0163)$

$\log_2 2 = 1$

$\frac{1}{4 \cdot \log_2 1.0163} = 10.72$ years

2. **Exponential Decay**
   A funny looking seashell was found in Lake Superior, and the NMU chemistry lab found that it contains 72% of the carbon-14 that is present in living seashells. Given that the half-life of carbon-14 is 5730 years, estimate the age of the seashell.

start: $A = Pe^{rt}$

$\frac{1}{2}P = Pe^{5730 \cdot r}$

$\ln\left(\frac{1}{2}\right) = 5730r \cdot \ln(e)$

$\frac{\ln(.5)}{5730} = r = R$

$.72P = Pe^{Rt}$

$\ln(.72) = Rt \ln(e) = Rt$

$\frac{\ln(.72)}{R} = \frac{\ln(.72)}{\left(\frac{\ln(.5)}{5730}\right)} = t = 2,715$ years

5. What is the relationship between

$$\left(1+\frac{1}{n}\right)^n$$

and the natural number e?

$\left(1+\frac{1}{n}\right)^n \longrightarrow e$ as $n \longrightarrow \infty$

6. **Polynomial and Rational Functions**
Find all the rational zeros of $f(x) = x^4 - 5x^3 + 6x^2 + 4x - 8$.

Start

$f(-1) = 1+5+6-4-8 = 0$

$f(2) = 16 - 40 + 24 + 8 - 8 = 0$

$(x+1)$ & $(x-2)$ are both factors. Thus $(x+1)(x-2)$ is a factor

$= (x^2 - x - 2)$

Next

$$x^2 - x - 2 \;\overline{)\; x^4 - 5x^3 + 6x^2 + 4x - 8}$$

$$-(x^4 - x^3 - 2x^2)$$

$$-4x^3 + 8x^2 + 4x - 8$$

$$-(-4x^3 + 4x^2 + 8x)$$

$$4x^2 - 4 - 8$$

$x^2 - 4x + 4$

zeros: $x = -1, 2$

So $f(x) = (x^2 - x - 2)(x^2 - 4x + 4)$

$(x-2)(x-2)$

All Rational zeros are: $-1, 2$

7. **Complex Numbers**
Evaluate and write in the form $a + bi$ the following

(a)

$$\frac{1}{1+i}\left(\frac{1-i}{1-i}\right) = \frac{1-i}{1+1} = \frac{1}{2} - \frac{1}{2}i$$

(b)

$$(1+2i)(3-4i) = 3 - 4i + 6i - 8i^2 = 11 + 2i$$

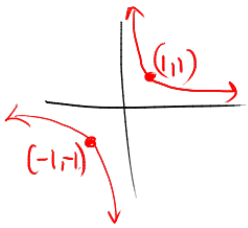
(c)

$$(2+3i)(2-3i) = 4 - 9i^2 = 13$$

(d)

$$i^{203} = i^{200} \cdot i^3 = 1 \cdot i^3 = -i$$

8. **Graphing and Interpreting Rational Functions**
Sketch a graph, determine all asymptotes, and all zeros of (a)



$$f(x) = \frac{1}{x}$$

V.A. @ $x = 0$
H.A @ $y = 0$

zeros: none

(b)

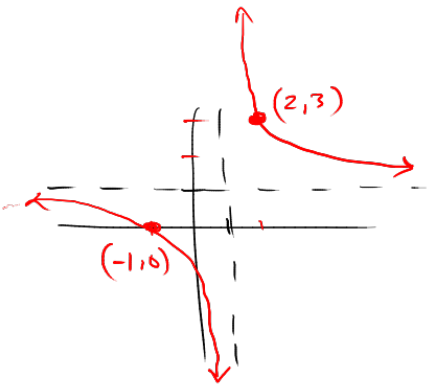
$$g(x) = \frac{x+1}{x-1}$$

$$x-1 \overline{)\, x+1} \quad \Rightarrow \quad g(x) = 1 + \frac{2}{x-1}$$

$$-(x-1)$$
$$2$$

H.A. @ $y = 1$
V.A @ $x = 1$

zeros: $x = -1$



@ $x = 3.5$, $g(x): \frac{5 \cdot (+)}{\oplus \ominus} < 0$

(c)

$$h(x) = \frac{5x + 10}{x^2 - 7x + 12} = \frac{5(x+2)}{(x-3)(x-4)}$$

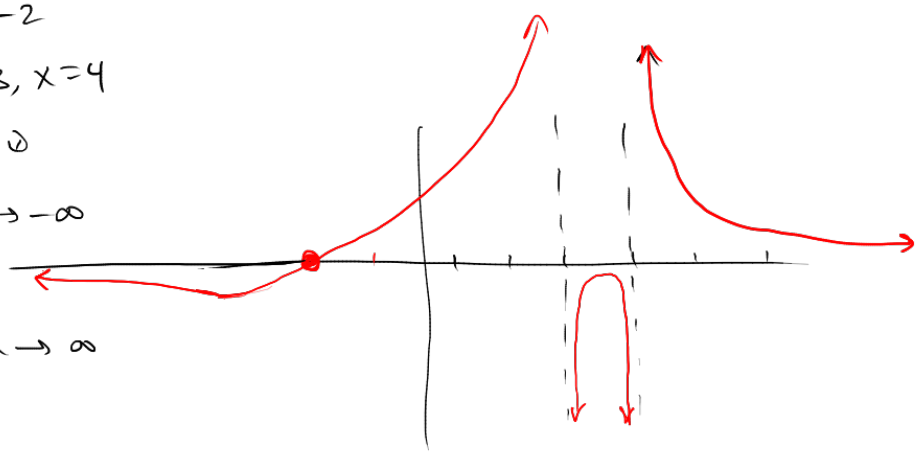
zeros: $x = -2$

V.A.: $x = 3, x = 4$

H.A.: $x = 0$

$h(x) < 0$ as $x \to -\infty$

$h(x) > 0$ as $x \to \infty$



(d)

$$k(x) = \frac{x^2 - 4x - 5}{x^2 - 6x - 16} = \frac{(x-5)(x+1)}{(x-8)(x+2)}$$

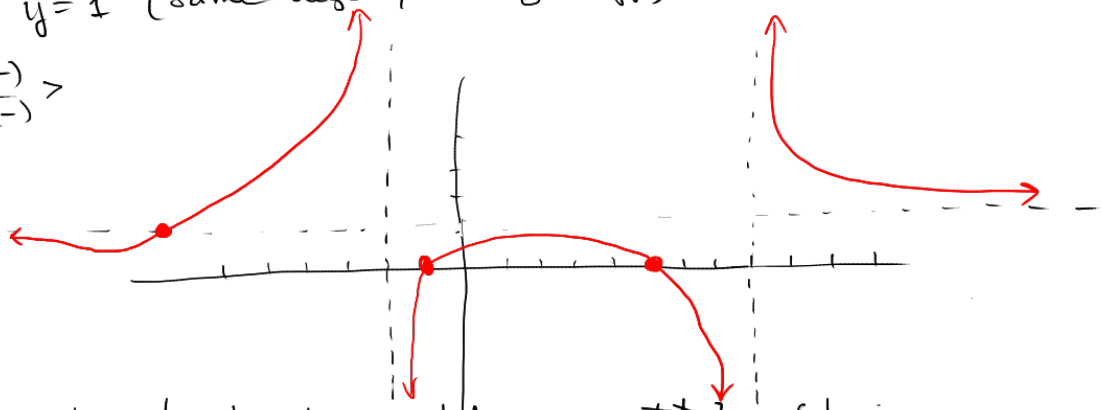
zeros: $x = 5, -1$

V.A. @ $x = 8, -2$

H.A @ $y = 1$ (same degree / leading coeffs)

$k(-3): \frac{(-)\cdot(-)}{(-)\cdot(-)} >$

$k(9): \frac{(+)\cdot(+)}{(+)\cdot(+)}$



Does $g(x)$ intersect its horizontal asymptote? false:

$$\frac{x^2 - 4x - 5}{x^2 - 6x - 16} = 1 \Rightarrow x^2 - 4x - 5 = x^2 - 6x - 16$$

$$\Rightarrow 2x = -11$$

$$\Rightarrow x = -11/2 = -5.2$$ is only intersection