$$\ln(x^2 - 2ex + e^2) = \pi$$

solve for x.

this is the outermost function affecting "x".

To solve – undo this

$$\boxed{e^{(\ln(x))} = x}$$

THE KEY

$$e^{\cancel{\ln}(x^2 - 2ex + e^2)} = e^{(\pi)}$$

$$x^2 - 2ex + e^2 = e^{\pi}$$

$$\quad\quad -e^{\pi} \quad -e^{\pi}$$

$$x^2 - 2ex + (e^2 - e^{\pi}) = 0$$

sq. lin. #

?

sq. lin. #

$$x^2 - 14x + 217 = 0$$

quadratic formula

$$x = \frac{-(-2e) \pm \sqrt{(-2e)^2 - 4(1)(e^2 - e^{\pi})}}{2 \cdot 1}$$

stop here.

or estimate using calculator.

MA115 :: Section 4.5 :: Exponential Functions and Modeling

$r > 0$

Exponential Growth: $n(t) = n_0 e^{rt}$ when $t=0$ $n(t) = n_0$ = "n - naught" = "initial population"

Radioactive Decay: $m(t) = m_0 e^{-rt}$

1. Obtain the current populations and growth rates of the following entities. Assume the population grows exponentially and estimate the populations in 2050.

   (a) Michigan growth rate: .27% = .0027

   $P_0 \approx 9.$ million $P = 9e^{.0027(36)} \approx$ 10.2 million

   (b) The United States. 407 million

   $\frac{1.9 \times 10^9}{4 \times 10^8} = \frac{1.9 \times 10}{4} \approx \frac{20}{4} = 5 \Rightarrow$

   Model ⇒ India will be 5 times as large as U.S

   (c) India $P_0 = 1.25$ billion

   $r = 1.2\% = .012 \Rightarrow P = 1.25 \cdot e^{.012(35)}$

   $= 1.9$ billion

2. It is estimated that in 1935 there were 240,000 deer in the UP. By 1955, there were an estimated 430,000 deer in the UP. Assuming the population grew exponentially, produce an equation representing the number of deer in the UP since 1935. Use your formula to estimate the number of deer in 1993.

$\boxed{n(t) = n_0 e^{rt}}$

unknown unknown

2 of these unknowns needs 2 data points to determine

$n(58) = 240{,}000 \cdot e^{.029(58)}$

$= 1.29$ million (this did not actually happen)

Two Points

(0, 240,000) Initial Pop

(20, 430,000) @ 1955

Initial Population $n(0) = 240{,}000$

$n_0 = 240{,}000$ because $n(0) = 240{,}000$

$n(t) = 240{,}000 \cdot e^{rt}$

$n(20) = 430{,}000 = 240{,}000 \cdot e^{r(20)} =$

$\frac{430}{240} = e^{20 \cdot r}$

$\ln(43/24) = 20 \cdot r \cdot \underbrace{\ln(e)}_{=1}$

$r = \frac{\ln(43/24)}{20}$

$= \boxed{.029}$

start $.65\cancel{P} = \cancel{P}e^{rt}$

then $.65 = e^{rt}$ goal: find r.

$\ln(.65) = \ln(e^{rt})$

$-.43 = r \cdot t$

$\frac{-.43}{t} = r$

$\left(\frac{-.43}{5730}\right) = r$

$-7.5 \times 10^{-5} = r$

$A = Pe^{rt}$

current pop → A; initial pop. → P

MA115 :: Section 4.5 :: Exponential Functions and Modeling

3. A wooden artifact from an old sailboat is found near Lake Superior to contain 65% of the carbon-14 that is present in living trees. How long ago was the sailboat made? (The half-life of carbon-14 is 5730 years.)

   ↳ how long it takes for P to decompose down to .65P

4. Newton's Law of Cooling: $T(t) = T_s + D_0 e^{-kt}$ where D_0 is the inital difference between an object and its environment, and if the environment has temperature T_s.

5. What temperature will your coffee be if you leave an uninsulated cup of coffee outside in the UP for 30 minutes duringan average day in January?

6. pH log scale:
$$pH = -log[H^+]$$
where $[H^+]$ is the concentration of hydrogen ions measured in moles per liter.

   Find the hydrogen ion concentration of a standard beer (pH = 4.6) and mash (fermenting mixed grain and water pH = 5.6). Also, find the hydrogen ion concentration of pure water (pH = 7.3).

MA115 :: Section 4.5 :: Exponential Functions and Modeling

r = growth rate, $(.27\% = .0027)$

Exponential Growth: $n(t) = n_0 e^{rt}$

initial pop. $= n_0$ "n-naught" $= n(0)$

Radioactive Decay: $m(t) = m_0 e^{-rt}$

1. Obtain the current populations and growth rates of the following entities. Assume the population grows exponentially and estimate the populations in 2050.

   (a) Michigan

   $n_0 \approx 9$ million in 2015, $r = .0027 \Rightarrow P_{2050} = P(35) = 9e^{.0027 \cdot 35} = 10.9$ million

   (b) The United States.

   $n_0 = 318$ million, $r = .7\% = .007 \Rightarrow P_{2050} = 318 \cdot e^{.007 \cdot 35} = 408$ million

   (c) India

   $n_0 = 1.2$ billion, $r = 1.24\% \Rightarrow P_{2050} = 1.2e^{.0124 \cdot 35} = 1.9$ billion (billion)

How much larger is India's pop projected to be larger than the U.S.

$\boxed{\dfrac{1.9 \times 10^9}{4 \times 10^8} = 5 \text{ Roughly Times}}$

2. It is estimated that in 1935 there were 240,000 deer in the UP. By 1955, there were an estimated 430,000 deer in the UP. Assuming the population grew exponentially, produce an equation representing the number of deer in the UP since 1935. Use your formula to estimate the number of deer in 1993.

$n(t) = n_0 \cdot e^{rt}$, t: years since 1935, n_0: Pop @ 1935

Two Data Points $(t, n(t))$: $(0, 240000)$, $(20, 430000)$

$\Rightarrow n(0) = 240000 \cdot e^{r \cdot 0} = 240{,}000 = n_0$, $n(t) = 240{,}000\, e^{rt}$

$\Rightarrow n(20) = 430000 = 240000\, e^{r \cdot 20}$ — to solve for r.... hit it with a log

$\frac{430}{240} = e^{20 \cdot r} \Rightarrow \ln\left(\frac{43}{24}\right) = \ln\left(e^{20 \cdot r}\right) = 20 \cdot r \Rightarrow r = \ln(43/24)/20 = .0292$

So, $n(t) = 240000\, e^{.0292 \cdot t}$

$n(58) = 240{,}000\, e^{.0292 \cdot 58} = 1.3$ million deer! this never actually happened

$A = Pe^{rt}$ → rate → time since initial amount was sampled
P → initial amount
A → current amount

$\ln(\frac{1}{2}) = \ln(1) - \ln(2)$
$= -\ln(2)$

use the half life like this:

$\frac{1}{2}P = Pe^{r \cdot 5730}$

$t = 5730$ means $A = \frac{1}{2}P$

rate of decay, given 1/2-life

3. A wooden artifact from an old sailboat is found near Lake Superior to contain 65% of the carbon-14 that is present in living trees. How long ago was the sailboat made? (The half-life of carbon-14 is 5730 years.)

Now solve for r.

$\ln(.5) = \ln(e^{5730 \cdot r}) = 5730 \cdot r$, so $r = \ln(.5)/5730$

$\boxed{r = -\ln(2)/5730}$

$10^\circ + \frac{120}{e^{-}} \to 0$

4. Newton's Law of Cooling: $T(t) = T_s + D_0 e^{-kt}$ where D_0 is the inital difference between an object and its environment, and if the environment has temperature T_s.

Formula

$A = Pe^{rt}$

$.65P = Pe^{\left(\frac{-\ln(2)}{5730}\right) \cdot t}$

$.65 = e^{-1.2 \times 10^{-4} \cdot t}$

$\ln(.65) = -1.2 \times 10^{-4} t \cdot \ln(e)$

$\ln(e) = 1$

$D_0 = 130^\circ - 10^\circ$
$= 120^\circ$

$3{,}600 \text{ years} = \frac{\ln(.65)}{-1.2 \times 10^{-4}} = t$

5. What temperature will your coffee be if you leave an uninsulated cup of coffee outside in the UP for 30 minutes duringan average day in January?

6. pH log scale:

$$pH = -log[H^+]$$

where $[H^+]$ is the concentration of hydrogen ions measured in moles per liter.

Find the hydrogen ion concentration of a standard beer (pH = 4.6) and mash (fermenting mixed grain and water pH = 5.6). Also, find the hydrogen ion concentration of pure water (pH = 7.3).

MA115 :: Section 4.5 :: Exponential Functions and Modeling

Exponential Growth: $n(t) = n_0 e^{rt}$

Radioactive Decay: $m(t) = m_0 e^{-rt}$

1. Obtain the current populations and growth rates of the following entities. Assume the population grows exponentially and estimate the populations in 2050.

   (a) Michigan

   (b) The United States.

   (c) India

2. It is estimated that in 1935 there were 240,000 deer in the UP. By 1955, there were an estimated 430,000 deer in the UP. Assuming the population grew exponentially, produce an equation representing the number of deer in the UP since 1935. Use your formula to estimate the number of deer in 1993.

Start: $A = Pe^{rt}$. Half-life = 5730 means: when $t = 5730$, $A = \frac{1}{2}P$ (current = half original)
(P = original amt, A = current amt.

solve for r

$\frac{1}{2}\cancel{P} = \cancel{P}e^{r \cdot 5730}$

$\frac{1}{2} = e^{5730 \cdot r}$

$\underbrace{\ln(\frac{1}{2})}_{-\ln(2) = -.69} = \ln(e^{5730r}) = 5730 \cdot r \cdot \underbrace{\ln(e)}_{=1}$

So: $\frac{-.69}{5730} = \boxed{r = -1.2 \times 10^{-4}}$

$\ln(\frac{1}{2}) = \underbrace{\ln(1)}_{0} - \ln(2)$

Update: $A = Pe^{(-1.2 \times 10^{-4})t}$. We want when A = 65% of P.

$.65\cancel{P} = \cancel{P}e^{(-1.2 \times 10^{-4})t} = (e^{-1.2 \times 10^{-4}})^t$

$e^{rt} = (e^r)^t$

$[.65 = (.999)^t]$

$\ln(.65) = t \ln(.999)$

$\ln(.65)/\ln(.999) = \boxed{t = 430 \text{ years.}}$

STO store $-1.2 \times 10^{-4} \to x$

$.65 = e^{-1.2 \times 10^{-4} \cdot t}$

$\ln(.65) = \ln(e^{-1.2 \times 10^{-4} \cdot t})$

$= -1.2 \times 10^{-4} \cdot t \cdot \underbrace{\ln(e)}_{1}$

3,580 years $= \frac{\ln(.65)}{-1.2 \times 10^{-4}} = t$

3. A wooden artifact from an old sailboat is found near Lake Superior to contain 65% of the carbon-14 that is present in living trees. How long ago was the sailboat made? (The half-life of carbon-14 is 5730 years.)

4. Newton's Law of Cooling: $T(t) = T_s + D_0 e^{-kt}$ where D_0 is the inital difference between an object and its environment, and if the environment has temperature T_s.

5. What temperature will your coffee be if you leave an uninsulated cup of coffee outside in the UP for 30 minutes duringan average day in January?

6. pH log scale:
$$pH = -log[H^+]$$
where $[H^+]$ is the concentration of hydrogen ions measured in moles per liter.

   Find the hydrogen ion concentration of a standard beer (pH = 4.6) and mash (fermenting mixed grain and water pH = 5.6). Also, find the hydrogen ion concentration of pure water (pH = 7.3).

MA115 :: Section 4.5 :: More Exponential Modeling

1. What is the half-life of a radioactive substance?

2. What is the doubling-time of a population?

3. Suppose that a culture contains 1500 bacteria initially and doubles every 30 min.

   (a) Find a function that models the number of bacteria $n(t)$ after t minutes.

   (b) Find the number of bacteria after 2 hours.

   (c) After how many minutes will the culture contain 4000 bacteria?

4. Carbon-14 dating is currently being used to monitor illegal elephant poaching. Recently, a shipment of 5 tons of ivory tusks was found in West Phillipines. Suppose a sample was found to have $\boxed{\%99.9}$ of the carbon-14 that is found in the tusks of living elephants. Approximately when was the elephant poached?

$A = Pe^{rt}$

Half-Life

$\frac{1}{2}P = Pe^{r(5730)}$

$\frac{1}{2} = e^{5730 \cdot r}$

$-\ln(2) = 5730 \cdot r \cdot \ln(e)$

$\frac{-\ln(2)}{5730} = r$

$.999 = e^{\left(\frac{-\ln(2)}{5730}\right) \cdot t}$

$\ln(.999) = \left(-\frac{\ln(2)}{5730}\right) t \cdot \underbrace{\ln(e)}_{=1}$

8.8 years $\approx \dfrac{\ln(.999)}{\left(-\frac{\ln(2)}{5730}\right)} = t$

$$\left[\dfrac{\ln(.999)}{\frac{-\ln(2)}{\text{Half Life}}}\right]$$

$$e^{2x} + e^x - 12 = 0$$

this is similar.

$w = e^x$. This becomes:

$w^2 = e^{2x}$

Trick: How do you solve

$$w^2 + w - 12 = 0$$

$$(w+4)(w-3) = 0$$

$w = -4$

or

$w = 3$

$$(e^x + 4)(e^x - 3) = 0$$

$e^x = -4$ $e^x = 3$

no solutions $\underset{x}{\underset{\shortparallel}{\ln(e^x)}} = \ln(3)$

never hits $y = -4$

$$\underbrace{\ln\left(e^{2x} + e^x - 12\right)}_{\text{doesn't simplify}} = \ln(0)$$

#23

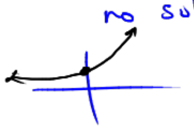
$$x^2 \cdot 2^x - 2^x \cdot 7 = 0$$

$$2^x(x^2 - 7) = 0$$

set each factor $= 0$

$2^x = 0$ $x^2 - 7 = 0$

no sol'n $x = \pm\sqrt{7}$



Exam 3 Study Guide :: Math 115 :: Winter 2015

1. **Exponential Functions**
   How long will it take for an investment of \$1000 to double in value if the interest rate is 6.5%, compounded quarterly?

2. **Exponential Decay**
   A funny looking seashell was found in Lake Superior, and the NMU chemistry lab found that it contains 72% of the carbon-14 that is present in living seashells. Given that the half-life of carbon-14 is 5730 years, estimate the age of the seashell.

3. **Logarithmic Models**
   On Tuesday of this week yet another earthquake occured in Oklahoma, this time of magnitude 4.0. In 2011, an earthquake of magnitude 9.0 occured off the coast of Japan, triggering a devastating tsunami. How many times more intense was the 2011 earthquake near Japan than this week's earthquake in Oklahoma.

4. **Algebra of Logarithmic and Exponential Functions**

   Simplify $\ln(e(x^2+1))$

   Simplify $\ln(x-1) - 3\ln(x+1) + \ln(e^x+1)$

   Solve $\log(x+1) - \log(x-1) = 2$

   Solve $\ln(x^2 - 5x - 23) = 0$

5. What is the relationship between

$$\left(1+\frac{1}{n}\right)^n$$

   and the natural number e?

6. **Polynomial and Rational Functions**
   Find all the rational zeros of $f(x) = x^4 - 5x^3 + 6x^2 + 4x - 8$.

7. **Complex Numbers**
   Evaluate and write in the form $a + bi$ the following
   (a)
$$\frac{1}{1+i}$$
   (b)
$$(1+2i)(3-4i)$$
   (c)
$$(2+3i)(2-3i)$$
   (d)
$$i^{203}$$

8. **Graphing and Interpreting Rational Functions**
   Sketch a graph, determine all asymptotes, and all zeros of (a)
$$f(x) = \frac{1}{x}$$

   (b)
$$g(x) = \frac{x+1}{x-1}$$

   (c)
$$h(x) = \frac{5x+10}{x^2-7x+12}$$

   (d)
$$k(x) = \frac{x^2-4x-5}{x^2-6x-16}$$

$$\log\left(\log\left(10000^{\boxed{10000}}\right)\right) = \underline{\quad 4 \quad} + \log(10000)$$

$$\log\left(10000 \cdot \log(10000)\right)$$

$$\log_{10}(\overset{=x}{10000}) + \log\left(\overline{\log(10000)}^{=4}\right)$$

$$10^x = 10000 = 10^4$$

$$x = 4$$

1. What is the half-life of a radioactive substance?

amount of time required for such a substance to reduce by a half

2. What is the doubling-time of a population?

time required for the population to double

3. Suppose that a culture contains 1500 bacteria initially and doubles every 30 min. $t = .5$ means $A = 2P$. $P =$ original amount

current amount

start:

$* \; A = Pe^{rt}$

15	0
30	.5
60	1
120	1.5
240	2

(a) Find a function that models the number of bacteria $n(t)$ after t minutes.

Use the doubling time:

$2P = Pe^{r(.5)}$

$$\boxed{n(t) = 1500e^{2\ln(2)t}}$$

$2 = e^{.5r}$

$$\ln(2) = \ln(e^{.5r}) = .5r \cdot \underbrace{\ln(e)}_{=1} = .5r \text{ so } r = \frac{\ln(2)}{.5} = 2\ln 2$$

(b) Find the number of bacteria after 2 hours.

$$n(2) = 1500e^{2\ln(2)\cdot 2} = \boxed{24{,}000}$$

$$= 1500 \cdot e^{\ln 2^2 \cdot 2} = 1500\left(\cancel{e}^{\ln(4)}\right)^2 = 1500 \cdot 4^2 = 1500 \cdot 16$$

(c) After how many minutes will the culture contain 4000 bacteria?

$$n(t) = 1500e^{\ln 4 t} = 1500 \cdot 4^t$$

$$4000 = 1500 \cdot 4^t$$

$$\frac{4000}{1500} = 4^t \quad \text{Hit it w/ ln} \quad \ln\left(\frac{8}{3}\right) = \ln(4^t) = t \cdot \ln(4)$$

$$\boxed{40 \text{ min}} \approx .707 \text{ hours} = \frac{\ln(8/3)}{\ln(4)} = t$$

4. Carbon-14 dating is currently being used to monitor illegal elephant poaching. Recently, a shipment of 5 tons of ivory tusks was found in West Phillipines. Suppose a sample was found to have %99.9 of the carbon-14 that is found in the tusks of living elephants. Approximately when was the elephant poached?

$A = Pe^{rt}$

- when $t = 5730$, set $A = \frac{1}{2}P$

$\frac{1}{2}\cancel{P} = \cancel{P}e^{5730 \cdot r}$

$\ln(\frac{1}{2}) = \overset{=0}{\cancel{\ln 1}} - \ln(2)$

$-\ln(2) = 5730r \cdot \underbrace{\ln(e)}_{=1}$

$\boxed{\frac{-\ln 2}{5730} = r}$

- $.999\cancel{P} = \cancel{P}e^{\left(\frac{-\ln 2}{5730}\right)t}$

$\ln(.999) = (-1.2 \times 10^{-4}) \cdot t \cdot \overbrace{\ln(e)}^{=1}$

$\frac{\ln(.999)}{(-1.2 \times 10^{-4})} \approx 8.3$ years compute.

$$\log(\log(10000^{1000}))$$

$$\log(1000 \cdot \underbrace{\log(\overbrace{10000}^{10^4})}_{=4}))$$

$$\log(1000 \cdot 4) = \log(4000)$$

$$\log(\underbrace{1000}_{10^3}) + \log 4$$

$$\boxed{\log_{10} 10^3} + \log 4 = 3 + \log 4$$

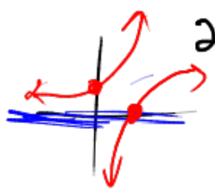
)

Solve:

$$2^x \cdot x^2 - 4 \cdot 2^x = 0$$

$$2^x(x^2 - 4) = 0$$

the product of two terms equals zero means that at least one of the factors is 0



$$2^x = 0$$

$$\log_2 2^x = \log_2 0$$

$$x = \log_2 0$$

(no sols here)

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$\boxed{x = \pm 2}$$

solve

$$23 - \ln(x-4) = 0$$

$$23 = \ln(x-4)$$

raise to power of e

$$e^{23} = e^{\ln(x-4)} = x-4$$

$$e^{23} + 4 = x$$

$18 - \ln(2 - x) = 0$

$$18 = \ln(2-x)$$

$$e^{18} = 2-x$$

$$x = 2 - e^{18}$$