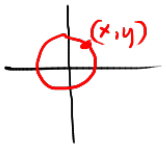


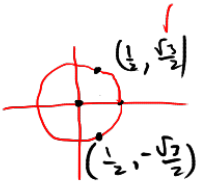
5.1 Trigonometry

MA115 :: Section ~~4.5~~ :: ~~More Exponential Modeling~~

The unit circle is the circle of radius 1 centered at the origin in the xy-plane and is given by

$$x^2 + y^2 = 1$$

1. Find the point(s) on the unit circle given by $(.5, y)$.



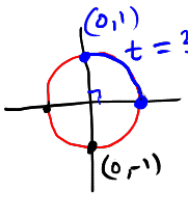
$(.5)^2 + y^2 = 1$

$y^2 = 1 - \frac{1}{4} = \frac{3}{4} \Rightarrow y = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$

2. Terminal Points on the Unit Circle.

Circumference of unit circle: $C = 2\pi r = 2\pi = 360°$

$\Rightarrow \pi = 180°$



Terminal Points for

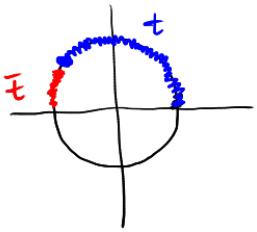
$\pi \hookrightarrow (-1,0)$

$\frac{3\pi}{2} = \frac{2\pi}{2} + \frac{\pi}{2} = (0,-1)$

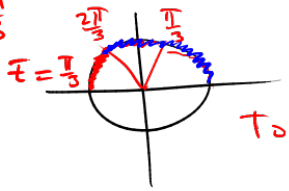
$3\pi - 2\pi + \pi = \pi$ $\quad (-1,0)$

3. Reference Number $\hat{t}$.

(smaller than $t \in \mathbb{R}$).



$t = \frac{2\pi}{3}$

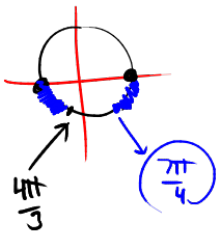


$\frac{\pi}{3} = \frac{180}{3} = 60$

To find $\hat{t}$: subtract from π or 2π

$t = \frac{2\pi}{3}$ think $\frac{2}{3} < 1$ so use π

$\pi - \frac{2\pi}{3} = \frac{\pi}{3}$

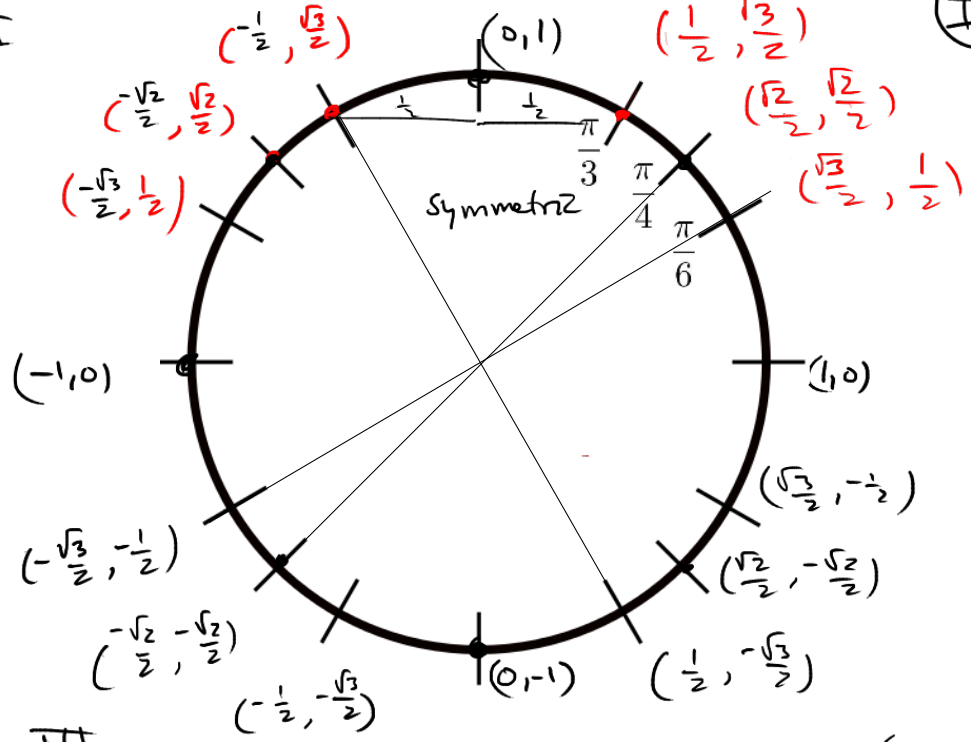


$2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$

$t = \frac{4\pi}{3} = \frac{3\pi}{3} + \frac{\pi}{3}$ what quadrant?

$\pi - \frac{4\pi}{3} = \left|-\frac{\pi}{3}\right| = \frac{\pi}{3} = \hat{t}$

$x^2+y^2=1$

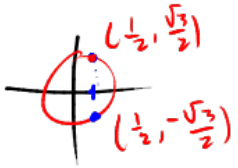


(−,+) II

I (+,+)

III (−,−)

IV (+,−)



MA115 :: Section 4.5 :: More Exponential Modeling

The unit circle is the circle of radius 1 centered at the origin in the xy-plane and is given by

$$x^2 + y^2 = 1$$

1. Find the point(s) on the unit circle given by $(.5, y)$.

$(.5)^2 + y^2 = 1$

$\frac{1}{4} + y^2 = 1 \Rightarrow y^2 = 1 - \frac{1}{4} = \frac{3}{4}$

$y = \pm\frac{\sqrt{3}}{2}$

2. **Terminal Points on the Unit Circle**: Start at (1,0), travel a distance t counterclockwise along the unit circle. You land at a terminal point (x,y) determined by t.

circumference: $C = 2\pi r = 2\pi$ = total distance around

$t = \pi$ (halfway around)

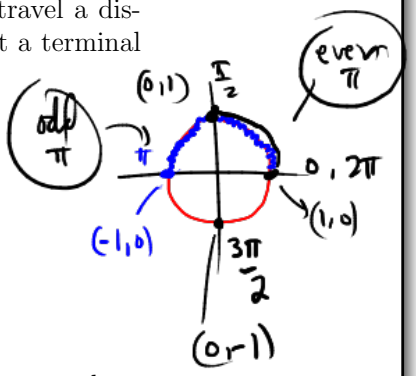
$t = \frac{\pi}{2} \leftrightarrow (0,1)$

$t = \frac{3\pi}{2} \leftrightarrow \frac{2\pi}{2} + \frac{\pi}{2} = \pi + \frac{\pi}{2}$

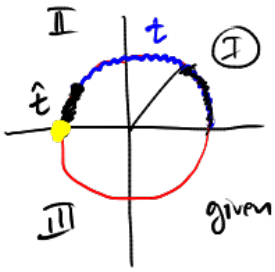
$t = -\frac{\pi}{2} \leftrightarrow (0,-1)$

$t = 3\pi \leftrightarrow (-1,0)$ (odd)

$t = 150\pi \leftrightarrow (1,0)$



3. **Reference Number** $\hat{t}$. Given $t \in \mathbb{R}$, $\hat{t}$ is the shortest distance along the unit circle between the t's terminal point and the x-axis.



Subtract From π (quadrant II, III)

From 2π (quadrant I IV)

given $\frac{5\pi}{6}$, in quadrant II, $\pi - \frac{5\pi}{6} = \frac{\pi}{6}$

given $\frac{7\pi}{4}$, in quadrant IV, $2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$

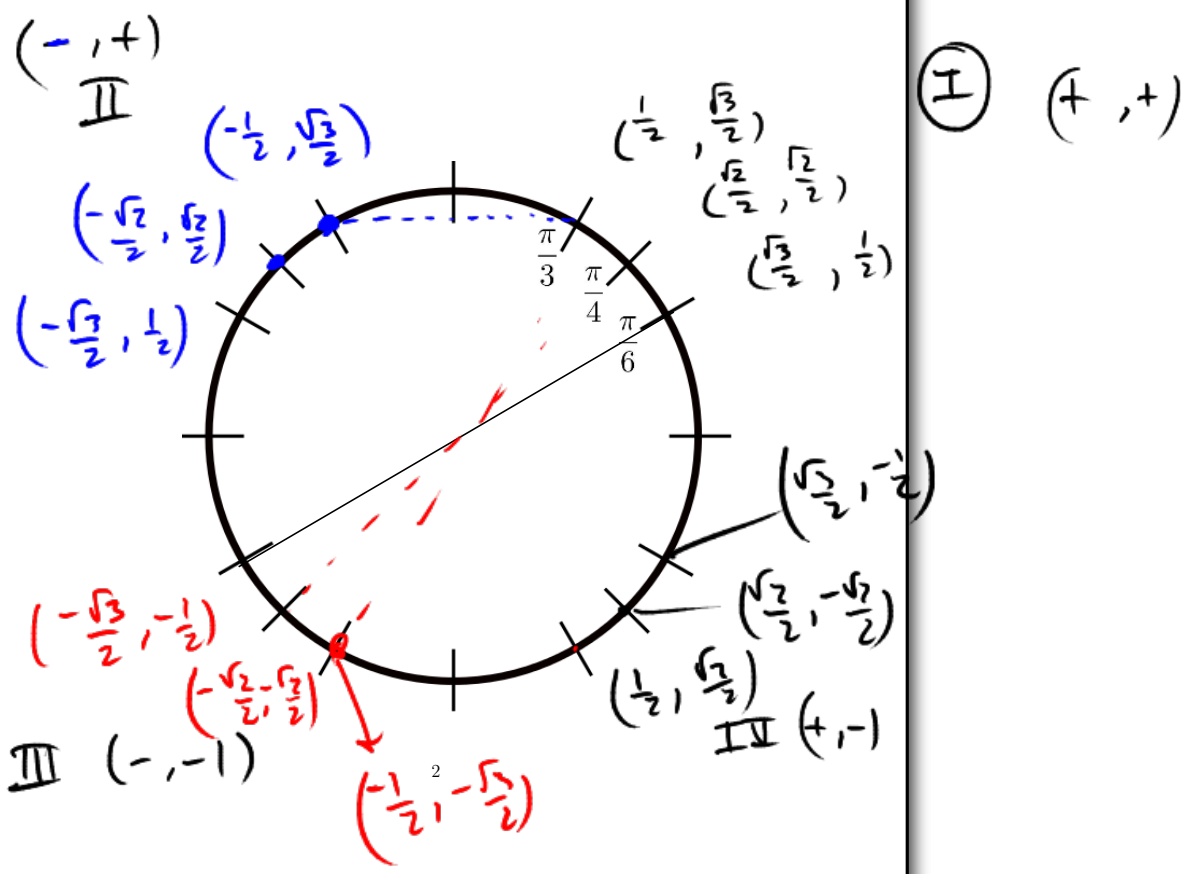
$= 2\pi - \frac{\pi}{4}$ — reference #

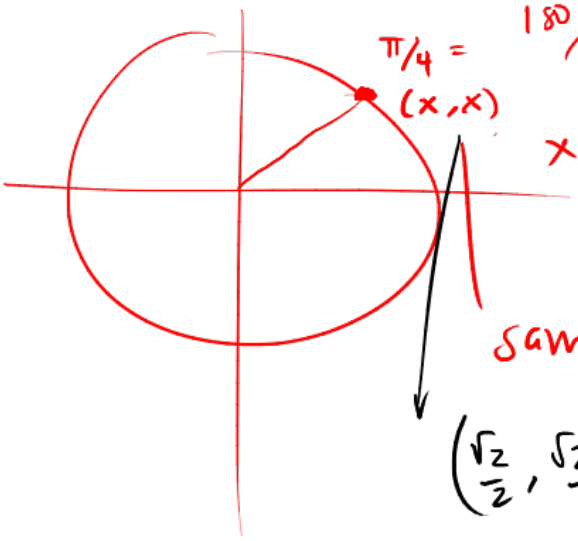
given $\frac{13\pi}{3} = 4\pi + \frac{\pi}{3}$ (only important)

$\frac{\pi}{3}$ I-quadrant

$\Rightarrow (1,0)$

4. Find all the indicated terminal points. Learn to do this by heart.





$x^2 + x^2 = 1$

$2x^2 = 1$

$x^2 = 1/2 \Rightarrow x = \pm \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\pm \frac{\sqrt{2}}{2}$

same b/c $45° \longleftrightarrow$ same over as up

MA115 :: Section 5.2 :: Trig Functions of Real Numbers

Warm-up For any real number t, what's the definition of $\sin t$ and $\cos t$ and the other standard trig functions?

$\sin(t)$ = y-coord. of the terminal point determined by t, $\csc(t) = \frac{1}{\sin(t)}$

$\cos(t)$ = x-coord of the terminal point determined by t $\sec(t) = \frac{1}{\cos(t)}$

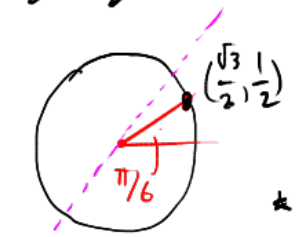
$\tan(t) = \frac{\sin(t)}{\cos(t)}$ think $\frac{y}{x}$ = slope 

, $\cot(t) = \frac{1}{\tan(t)}$

$\pi = 180^\circ$, $\pi/6 = 30^\circ$

$\pm\frac{1}{2}, \pm\frac{\sqrt{3}}{2}$

1. Fill in the table of special values of the standard trig functions



vertical line has ∞-slope

$\frac{1}{0} \to \frac{0}{1}$

	$\frac{y}{\sin t}$	$\cos t$	$\frac{\sin}{\cos}$ $\tan t$	$\csc t$	$\sec t$	$\cot t$
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{2}$	1	0	DNE	1	DNE	0
	0	0	1	0	DNE	1	DNE

$\frac{2}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\frac{\pi}{4} = 45^\circ$ 

$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

$\frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$\frac{3}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}}\frac{2\sqrt{3}}{3}$

Function	$\sin(t)$	$\cos(t)$	$\tan(t)$	$\csc(t)$	$\sec(t)$	$\cot(t)$
Domain	$\mathbb{R}$	$\mathbb{R}$	$\mathbb{R}-\{\text{odd}\cdot\frac{\pi}{2}\}$ throw out these	$\mathbb{R}-\{n\cdot\pi \mid n\in\mathbb{Z}\}$ $(\sin(t)=0)$	$\mathbb{R}-\{\text{odd}\cdot\frac{\pi}{2}\}$ $\cos t=0$	$\mathbb{R}-\{n\cdot\pi \mid n\in\mathbb{Z}\}$ $(\sin(t)=0)$

MA115 :: Section 5.2 :: Trig Functions of Real Numbers

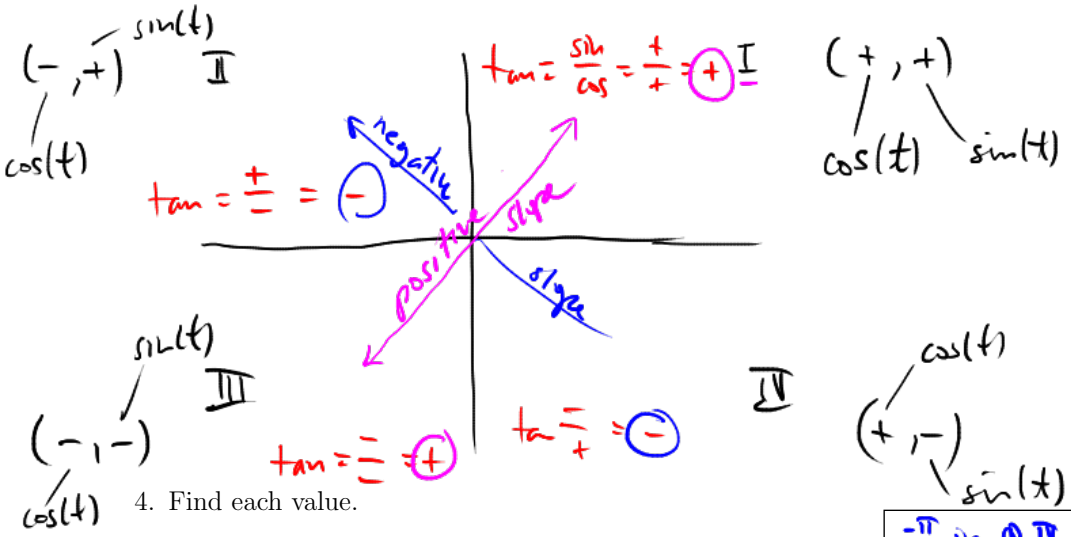
$\mathbb{Z}$ = set of integers, $\in$ = "belongs to"

2. What are the domains of the trig functions?

$\tan(t)=\frac{\sin(t)}{\cos(t)}$ so $\cos(t)$ can't be 0. This occurs at $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$ ($\cos(t)=0$ HERE)

$\sin(t)$ ○ | $\cot=\frac{1}{\tan}=\frac{1}{\frac{\sin}{\cos}}=\frac{\cos}{\sin}$

3. What are the signs of the trig functions in each quadrant?



4. Find each value.

(a) $\sin\frac{3\pi}{4}$ (b) $\cos\frac{25\pi}{3}$ (c) $\tan\left(-\frac{\pi}{4}\right)$

Ref #

$\frac{\pi}{4}$ $\frac{\pi}{3}$ $\frac{\pi}{4}$

$-\frac{\pi}{4}$ in QIV, $\tan(-\frac{\pi}{4})=\pm 1$

$\tan(t)<0$ in QIV

so $\tan(-\frac{\pi}{4})=-1$

tells you:

$\sin(\frac{3\pi}{4})=\pm\sin(\frac{\pi}{4})$

$=\pm\frac{\sqrt{2}}{2}$

$\frac{3\pi}{4}$ in QII, so use +

$\frac{\sqrt{2}}{2}$

$\cos(\frac{25\pi}{3})=\pm\cos(\frac{\pi}{3})$

$=\pm\frac{1}{2}$

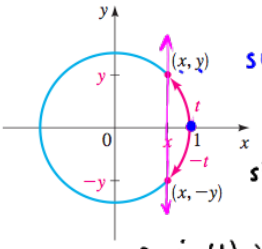
start ↺ (1,0)

$\frac{25\pi}{3}=\frac{24\pi}{3}+\frac{\pi}{3}=8\pi+\frac{\pi}{3}$

QI ⇒ $\cos(t)>0$

choose $\frac{1}{2}$

$\cos(t) = x = \cos(-t)$



$\sin(t) = y$

$\sin(-t) = -y$

$= -\sin t$

$\Rightarrow \sin(t)$ is odd

5. See the figure: $\sin x$ is odd and $\cos x$ is even.

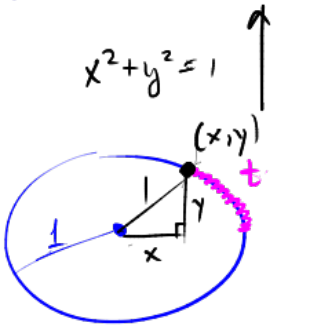
   Example: Compute $\sin\left(-\frac{\pi}{3}\right)$ and $\cos\left(-\frac{\pi}{3}\right)$

$-\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$(\cos t, \sin t)$

$x^2 + y^2 = 1$



6. TWO IMPORTANT IDENTITIES

$$\boxed{\tan t = \frac{\sin t}{\cos t}} \text{ and } \boxed{\sin^2 t + \cos^2 t = 1}$$

7. If $\cos t = -\frac{4}{5}$ and t is in quadrant III find the values of all the trig functions at t.

don't know $\sin(t)$

do know: $(\sin(t))^2 + (\cos(t))^2 = 1$

$(\sin t)^2 + (-4/5)^2 = 1$

$(\sin t)^2 = 1 - \frac{16}{25} = \frac{25}{25} - \frac{16}{25} = \frac{9}{25}$

$\sin t = \pm \frac{3}{5}$ choose neg. b/c Q III

$\sin t = -\frac{3}{5}$

$\tan t = \frac{\sin}{\cos} = \frac{-3/5}{-4/5} = 3/4$

$\cot t = 4/3$

$\csc t = \frac{1}{\sin} = \frac{-5}{3}$, $\sec t = -\frac{5}{4}$

8. Bungee jumping was once allowed on the New River Gorge bridge in West Virginia. The bridge is 876 feet tall and jumpers would plummet from the bridge down toward the river and then bounce back over and over again. At time t seconds after her jump a ladies height H (in feet) above the river is given by

$$H(t) = 400 + 476e^{-t/20}\cos\left(\frac{\pi}{4}t\right)$$

Find her height at time $t = \{0, 1, 2, 4, 6, 8, 10, 12, 16\}$.