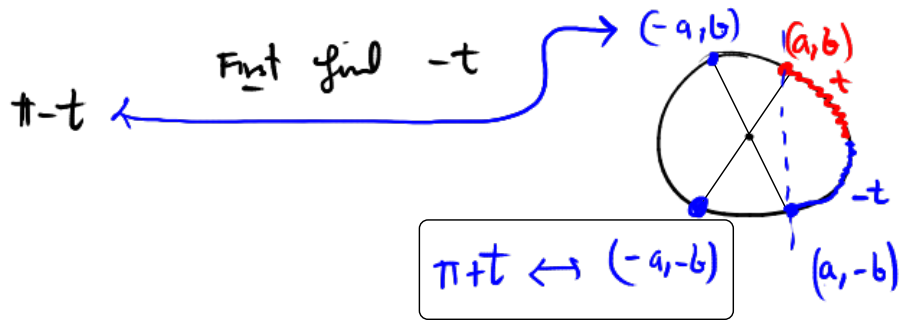
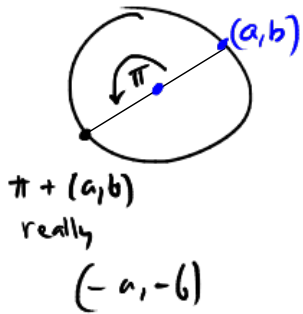
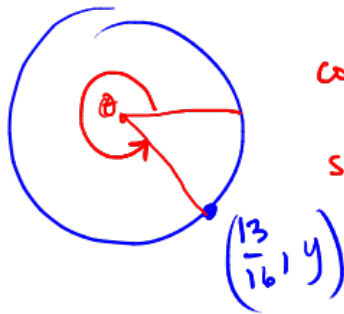


WebWork Questions?



5.1.4



$$\cos(\theta) = \frac{13}{16}$$

$$\sin \theta = y$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\frac{169}{256} + \sin^2 \theta = 1$$

$$\sin^2 \theta = 1 - \frac{169}{256} = \frac{87}{256}$$

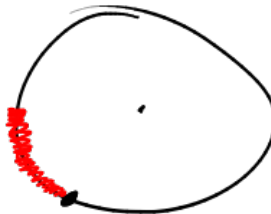
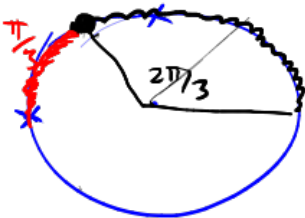
$$\sin \theta = \pm \sqrt{\frac{87}{256}}$$

\Rightarrow choose negative

5.1.3

Reference #

$$\frac{2\pi}{3} = \frac{2}{3} \times \pi$$



5.2.11

quadratic-type



just like:

$$w = \sin x$$

$$3w^2 - 7w + 2 = 0$$

$$(3w - 1)(w - 2) = 0$$

$$3w = 1 \text{ or } w = 2$$

$$\sin x = w = \frac{1}{3}$$

$$\text{or } w = 2$$

$$\sin x = 2$$

$$3 \sin^2 x - 7 \sin x + 2 = 0$$

$$\hat{\sin} x$$

$$x = \sin^{-1}\left(\frac{1}{3}\right) \text{ \& } x = \sin^{-1}(2)$$

$$2(\sin x)^2 - 5 \cos x - 4 = 0$$



$$2(1 - \cos^2 x) - 5 \cos x - 4$$

$$-2 \cos^2 x - 5 \cos x - 2 = 0$$

$$w = \cos x \Rightarrow -2w^2 - 5w - 2 = 0$$

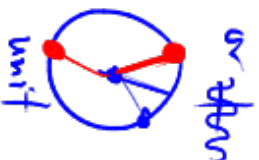
same as

$$2w^2 + 5w + 2 = 0$$

$$(2w + 1)(w + 2) = 0$$

$$2w = -1$$

$$w = -\frac{1}{2}, \quad w = -2$$

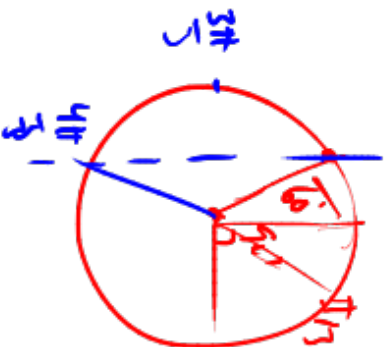


$$x = \frac{2\pi}{3} \text{ or } \frac{4\pi}{3}$$

$$\cos x = -\frac{1}{2}$$

$$\cos x = -2$$

no sol's

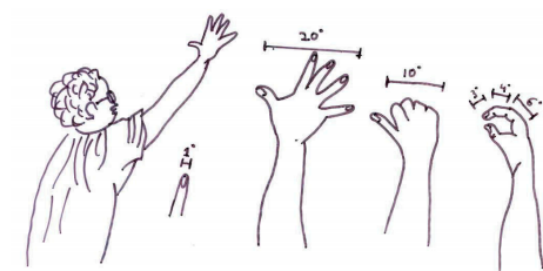


$$\frac{2\pi}{3}$$

MA115 :: Section 6.2 & 6.3 :: Right angle trigonometry

There are several methods of measuring angles of varying sophistication (eg., the iGradient app), but I'd like you to learn to use one rather unso-fisticated method. We will use our own body parts to measure angles.

This is how you do it - stretch out your arm. Your fist will cover an angle of about 10° , your index finger 1° and your open hand about 20° .

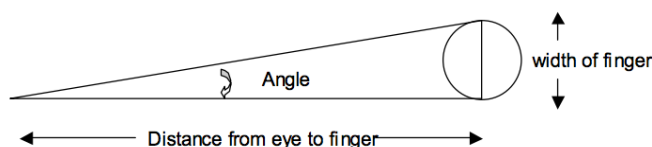


First, let's check this using some trig.

1. Extend your arm in front of you. Measure the distance between your eye and the tip of your finger.

2. Measure the width of your index finger. (across your finger nail.)

3. Calculate the angular width of your finger in degrees.



MA115 :: Section 6.2 & 6.3 :: Right angle trigonometry

Example. *Find the height of the old Jamrich Hall, but this time go outside and estimate the angle, height and distance yourself. Compare your answer to the previous one.*

Example. *Mesure the height of a light pole on academic mall.*

Example. *Time permitting, **estimate the height of the northeast corner of the new Jamrich Hall.** You'll need a partner to estimate the distance from the fence to the builing using your hands*

$$3\sin^2 x - 7\sin x + 2 = 0 \quad \text{ux. } \sin^{-1}$$

$$w = \sin x$$

$$3w^2 - 7w + 2 = 0$$

$$(3w - 1)(w - 2) = 0$$

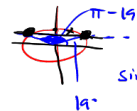
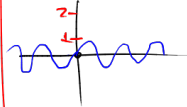
$$3w = 1 \quad \sin x = w = \frac{1}{3}$$

$$\sin x = w = \frac{1}{3}$$

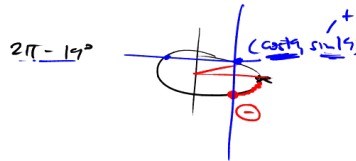
what's x?

$$x = \sin^{-1}\left(\frac{1}{3}\right), \sin^{-1}(2)$$

error.



$$\sin(\pi - 1.9) = \sin(1.9)$$



$$e^{2x} - e^x - 6 = 0$$

↓ quadratic type

$$(e^x - 3)(e^x + 2) = 0$$

(12)

$$2(\sin x)^2 - 5\cos x - 4 = 0$$

$$2(1 - \cos^2 x) - 5\cos x - 4 = 0$$

$$2 - 2\cos^2 x - 5\cos x - 4 = 0$$

$$-2\cos^2 x - 5\cos x - 2 = 0$$

clear neg signs

$$2\cos^2 x + 5\cos x + 2 = 0$$

similar to 11, but with the added step that we have BOTH $\sin(x)$ and $\cos(x)$. before we can proceed as in 11, we need get this down to just one trig function

need relationship b/w $\sin(x)^2$ and $\cos(x)$

$$\textcircled{*} (\sin^2 x + \cos^2 x = 1)$$

$$\sin^2 x = 1 - \cos^2 x$$

this factors easily, proceed as in 11

(13)

$$\tan^5 x - 9\tan x = 0.$$

$$\tan x (\tan^4 x - 9) = 0$$

$$\tan x = 0$$

$$\frac{\sin x}{\cos x} = 0 \quad \text{same sol's} \quad \sin x = 0$$

$$x = n \cdot \pi, n \text{ is any integer}$$

when a fraction/rational expression equals zero, the numerator must equal 0

$$\tan^4 x - 9 = (\tan^2 x - 3)(\tan^2 x + 3) = 0$$

$$\tan^2 x = 3$$

$$\tan x = \pm \sqrt{3}$$



$$\tan^2 x = -3$$

$$\tan x = \pm \sqrt{-3}$$

not real sol's.

$$\tan(120^\circ) = -\sqrt{3}$$

$$\tan(60^\circ) = \sqrt{3}$$

$$60^\circ = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \dots = Ak\pi$$

$$A = \frac{1}{3}, k \text{ is an integer}$$

6.2.7.

$$\sin \theta = \frac{3}{5}, \theta \text{ is acute}$$

find all trig functions of θ

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{3/5}{4/5} = \frac{3}{4}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\left(\frac{3}{5}\right)^2 + \cos^2 \theta = 1$$

$$\cos^2 \theta = 1 - \frac{9}{25} = \frac{16}{25}$$

$$\cos \theta = \frac{4}{5}$$

6.2.16

$$\cos \theta = -\frac{6}{12}, \theta \in \text{Quadr. III.}$$



$$\tan(\theta) \cot(\theta) = \frac{\tan \theta}{1} \cdot \frac{1}{\tan \theta} = \frac{\tan \theta}{\tan \theta} = 1$$

$$\csc \theta \cdot \tan \theta = \frac{1}{\cancel{\sin \theta}} \cdot \frac{\cancel{\sin \theta}}{\cos \theta} = \frac{1}{\cos \theta} = \frac{1}{-6/12} = -\frac{12}{6} = -2$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$6.4 \frac{1}{2} 6.5$$

Law of sines and cosines

when you have right triangle



$$180 - (30 + 90) = 60$$

find all lengths / angles

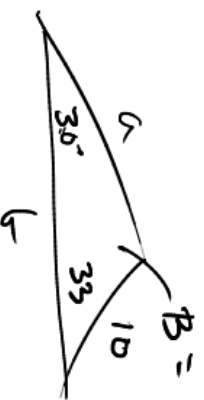
You can "solve" using basic trig definitions

if we don't ...

need

Law of Sines:

$$B = 180 - 63 = 127$$



know

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



$$\sin 30 = \frac{10}{c} \Rightarrow c = \frac{10}{\sin 30} = \frac{10}{1/2}$$

$$c = 20$$

$$\frac{\sin 30^\circ}{10} = \frac{\sin 33^\circ}{a}$$

$$a = \sin 33^\circ \left(\frac{10}{\sin 30^\circ} \right)$$

$$= 10.9$$

$$\frac{\sin 117}{b} = \frac{\sin 30}{10}$$

proceed as here

so, are there any questions?

5.2.13 -
webwork

$$\tan^5 x - 9 \tan x = 0.$$

$$\underbrace{\tan(x)}_{=0} (\underbrace{\tan^4(x) - 9}_{=0}) = 0$$

no real sols
 $\tan x = \sqrt{-3}$
 $\tan^2 x = -3$

$$\hookrightarrow (\tan^2 x - 3)(\tan^2 x + 3) = 0$$

$$\tan(x) = 0$$

$$\frac{\sin x}{\cos x}$$

means

$$\sin x = 0$$



$$\Rightarrow x = k\pi \text{ where } k \in \mathbb{Z}$$

$$A k \pi$$

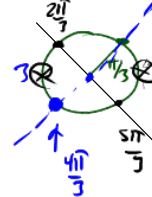
$$\frac{1}{3} k \pi = \frac{k \pi}{3}$$

Last case:

$$\tan^2 x = 3$$

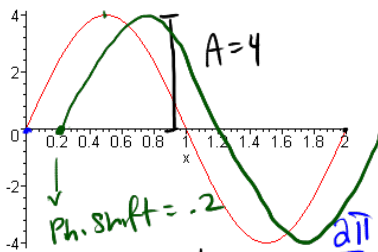
$$\tan x = \pm \sqrt{3}$$

$$\tan 60^\circ = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$$



any multiple
of $\frac{\pi}{3}$

5.3.10



$$\text{Period} = 2 = \frac{2\pi}{k}$$

$$4 \sin(\pi x)$$

Amplitude

Amplitude: $\frac{1}{2}$ distance from Crest to Trough
along y-axis
 $\sin x$: amplitude = 1

Period: length of x-values till graph repeats

Phase shift: horizontal translation

$$A \sin(kx)$$

$$A \sin(k(x-h))$$

phase shift

5.3.2 For

$$y = -9 \cos(x - \frac{\pi}{7})$$

$$A = |-9| = 9$$

$$\textcircled{1} \Rightarrow \text{period is } \frac{2\pi}{1} = 2\pi$$

phase shift

What about:

$$y = -5 \sin(3x - 9) = -5 \sin(3(x-3))$$

$$\text{Amp} = 5$$

$$\text{period} = \frac{2\pi}{3}$$

$$\text{Phase shift} = 3$$

5.3.7

$$y = \sin\left(2x + \frac{\pi}{2}\right) = \sin\left(2\left(x + \frac{\pi}{4}\right)\right) = \sin\left(2x + \frac{2\pi}{4}\right)$$

Amplitude = 1

Phase Shift = $-\frac{\pi}{4}$

$$\frac{\pi}{2} = \frac{2\pi}{2 \cdot 2}$$

Factor out of $\frac{\pi}{2}$ gives $\frac{\pi}{4}$

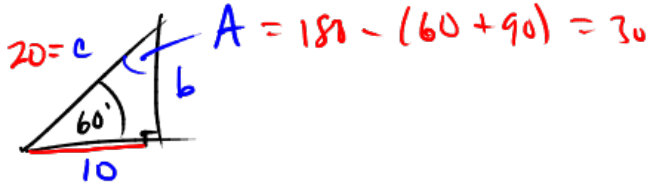
$$2x + \frac{2\pi}{4} = 2x + \frac{2\pi}{18}$$

$$= 2\left(x + \frac{\pi}{18}\right)$$

$$\hookrightarrow \text{Phase Shift} = -\pi/18$$

6.4 & 6.5 Law of Sines and Cosines

It's easy to solve right Δ 's

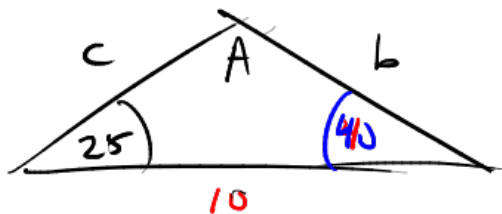


$$\cos(60^\circ) = \frac{\text{adj}}{\text{hyp}} = \frac{10}{c}$$

$$c = \frac{10}{\cos 60^\circ} = \frac{10}{(\frac{1}{2})} = 20$$

Pythag Thm \Rightarrow find b .

Use Law of sines . Solve Δ . Find A, b, c .



ASA

$$A = 180 - 65 = 115$$

$$\frac{\sin 25}{b} = \frac{\sin 115}{10}$$

$$b = \frac{\sin 25 \cdot 10}{\sin 115} = 4.6$$

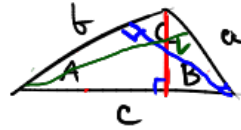
Repeat for c . $\frac{\sin 40}{c} = \frac{\sin 115}{10} \Rightarrow c = \frac{\sin 40 \cdot 10}{\sin 115} = 7.1$

Law of Cosines

sort of like pythagorean theorem.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

less Easy



$$\begin{aligned} A &= \frac{1}{2} b \cdot h \\ &= \frac{1}{2} c \cdot b \sin A \\ &= \frac{1}{2} b a \sin C \\ &= \frac{1}{2} a c \sin B \end{aligned}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

— know this —

$$\frac{2 \cdot \frac{1}{2} b c \sin A}{abc} = \frac{2 \cdot \frac{1}{2} a b \sin C}{abc} = \frac{2 \cdot \frac{1}{2} a c \sin B}{abc}$$

$$\frac{\sin A}{a} = \frac{\sin C}{c} = \frac{\sin B}{b}$$