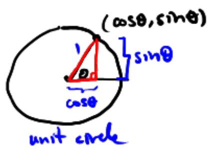
MA115 :: Section 6.2 & 6.3 :: Right angle trigonometry

> **Objective:** We will define the trigonometric functions in terms of raios of sides of right triangles and use them to solve practical problems.

For a right triangle with angle θ as one of its acute angles the trig ratios are defined as follows:

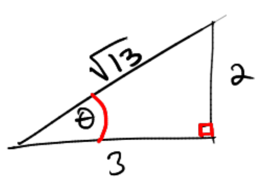


SOH CAH TOA

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ $\qquad \cos\theta = \dfrac{\text{adj}}{\text{hyp}}$ $\qquad \tan\theta = \dfrac{\text{opp}}{\text{adj}}$

$\csc\theta = \dfrac{\text{hyp}}{\text{opp}}$ $\qquad \sec\theta = \dfrac{\text{hyp}}{\text{adj}}$ $\qquad \cot\theta = \dfrac{\text{adj}}{\text{opp}}$

Example. *Find the six trig ratios of the angle θ of the following triangle:*



$\sin\theta = \dfrac{2}{\sqrt{13}} = \dfrac{2\sqrt{13}}{13}$ $\qquad \cos\theta = \dfrac{3}{\sqrt{13}} = \dfrac{3\sqrt{13}}{13}$ $\qquad \tan\theta = \dfrac{2}{3}$

use $a^2+b^2=c^2$
to find hyp
$3^2+2^2=13$

$\csc\theta = \dfrac{\sqrt{13}}{2}$ $\qquad \sec\theta = \dfrac{\sqrt{13}}{3}$ $\qquad \cot\theta = \dfrac{3}{2}$

Example. *If $\cos\theta = \sqrt{.74}$ find the other five trig ratios of θ.*

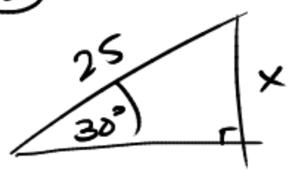
$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - .74 = .26 \Rightarrow \sin\theta = .51$

$\sin\theta = .51$ $\qquad \cos\theta = \sqrt{.74} = .86$ $\qquad \tan\theta = \dfrac{.51}{.86} = .593$

$\csc\theta = \dfrac{1}{.51} = 1.96$ $\qquad \sec\theta = \dfrac{1}{.86} = 1.16$ $\qquad \cot\theta = 1.69$

(3)



Find the length x.

$x =$ opp side of $30°$

$\frac{x}{25} = \frac{opp}{hyp} = \sin 30°$ cross multiply $x = 25 \cdot \sin 30°$

$= \frac{25}{2} = \underline{12.5}$

(4) $\csc\theta = \frac{50}{17}$

we want to know

$\sin\theta$ $\underline{17/50}$

$\tan\theta$ $\underline{.36}$

$\csc\theta = \frac{1}{\sin\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{17}{50}\right)^2 + \cos^2\theta = 1$

$\cos^2\theta = 1 - (17/50)^2 \Rightarrow \underline{\cos^2\theta = .8844}$

$\Rightarrow \cos\theta = .94$

$\tan\theta = \frac{17/50}{.94} = .36$

$=$

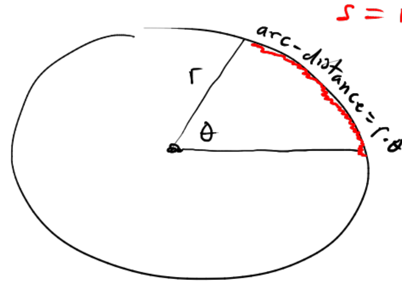
THE UNIT CIRCLE

this distance is θ

circumference: $2\pi \cdot 1 = 2\pi$

ANY CIRCLE

$s = r\theta$



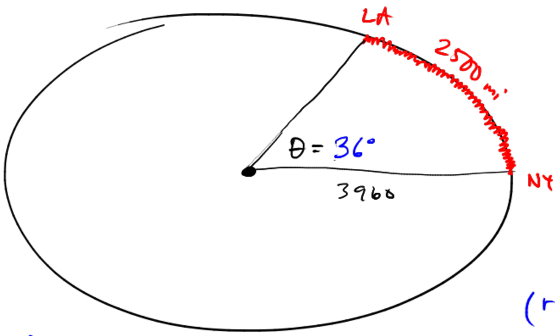
Radians ⟶ Degrees

$.63 \times \frac{180}{\pi} = 36°$

Earth

$r = 3960$ miles

$d(NY, LA) = 2500$ miles



$S = r \cdot \theta$ {always radians}

$2500 = 3960 \cdot \theta$

$.63 = \frac{2500}{3960} = \theta$

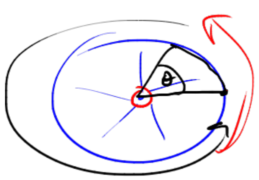
(radians)

EARTH'S CIRCUMFERENCE: 24,901 mi.

Verify the formula $S = r \cdot \theta$

$24{,}901 = 3960\,\theta$? $\Rightarrow \theta = \frac{24{,}901}{3960} = 360$

Circular Motion



bigger wheels ⟹ faster speed.

Linear speed: rate at which distance traveled changes

$V = \frac{s}{t} = \frac{\text{distance}}{\text{time}}$

Angular Speed: rate at which central angle changes

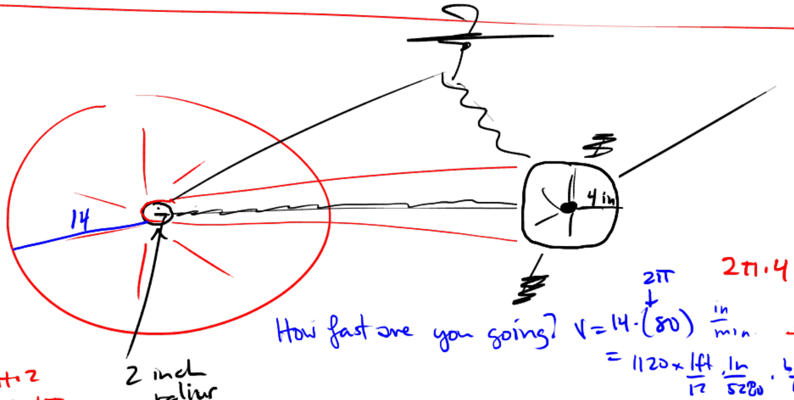
$\omega = \frac{\theta}{t}$

size doesn't matter

Remember $s = r\theta$

$V = r \cdot \omega$

$1 \text{ rev} = 2\pi$



$2\pi \cdot 2 = 4\pi$

Assume: You pedal at 40 RMPs

$2\pi \cdot 4 = 8\pi$

How fast are you going? $V = 14 \cdot (80)\, 2\pi \frac{\text{in}}{\text{min}}$ ⟹ Rear wheel spins @ 80 RPMs

$= 1120\pi \times \frac{1\text{ft}}{12\text{ in}} \cdot \frac{1\text{mi}}{5280\text{ ft}} \cdot \frac{60\text{ min}}{1\text{ hr}}$

$= 6.6$ m/H.

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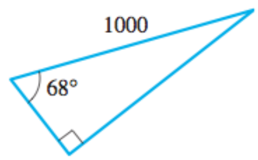
Example. *Solve the triangle*



Example. *The angle of elevation to the top of the old Jamrich Hall is found to be* 60° *from 5.5 feet above ground at a distance of 20 feet. Using this information find the height of the old Jamrich Hall.*

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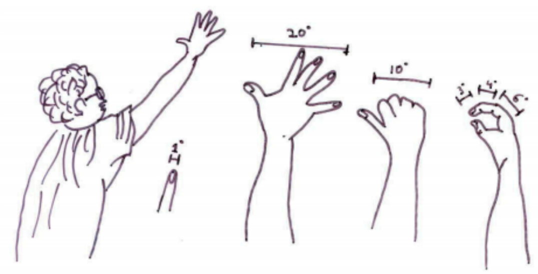
There are several methods of measuring angles of varying sophistication (eg., the iGradient app), but I'd like you to learn to use one rather unso-fist-icated method. We will use our own body parts to measure angles.

This is how you do it - stretch out your arm. Your fist will cover an angle of about 10°, your index finger 1° and your open hand about 20°.



First, let's check this using some trig.

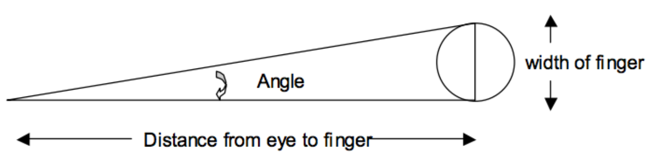
1. Extend your arm in front of you. Measure the distance between your eye and the tip of your finger.

   ____

2. Measure the width of your index finger. (across your finger nail.)

   ____

3. Calculate the angular width of your finger in degrees.



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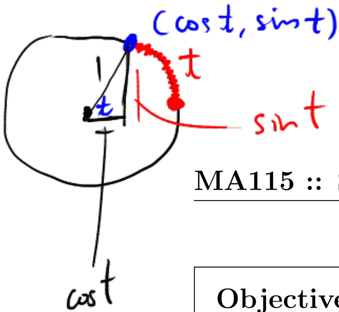
Example. *Find the height of the old Jamrich Hall, but this time go outside and estimate the angle, height and distance yourself. Compare your answer to the previous one.*

Example. *Mesure the height of a light pole on academic mall.*

Example. *Time permitting,* ***estimate the height of the northeast corner of the new Jamrich Hall****. You'll need a partner to estimate the distance from the fence to the builing using your hands*



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For a right triangle with angle θ as one of its acute angles the trig ratios are defined as follows:

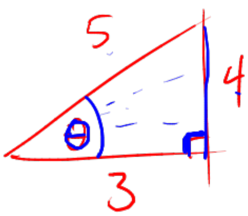
SOHCAHTOA

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$ $\qquad \cos\theta = \frac{\text{adj}}{\text{hyp}}$ $\qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$

inverse

$\csc\theta = \frac{\text{hyp}}{\text{opp}}$ $\qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$ $\qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$

Example. *Find the six trig ratios of the angle θ of the following triangle:*



$\sin\theta = \frac{4}{5}$ $\qquad \cos\theta = \frac{3}{5}$ $\qquad \tan\theta = \frac{4}{3}$

$3^2 + 4^2 = c^2$

$\csc\theta = \frac{5}{4}$ $\qquad \sec\theta = \frac{5}{3}$ $\qquad \cot\theta = \frac{3}{4}$

Example. *If* $\boxed{\cos\theta = \sqrt{.74}}$ *find the other five trig ratios of* θ.

$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - (\sqrt{.74})^2$

$\cos\theta$

$\boxed{\sin\theta = .51}$

$\sin\theta = .51$ $\qquad \cos\theta = \sqrt{.74}$ $\qquad \tan\theta = .51/\sqrt{.74} = .53$

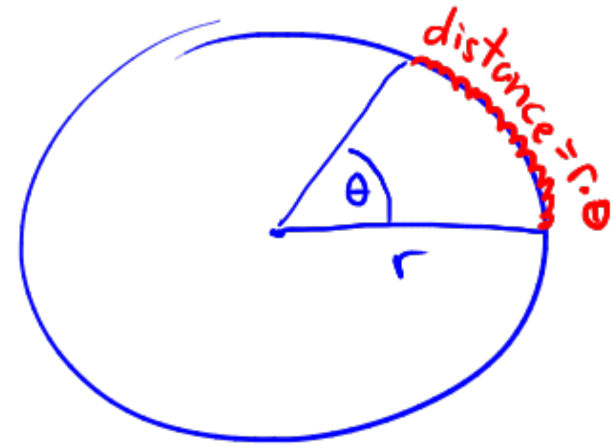
$\csc\theta = 1/.51 =$ $\qquad \sec\theta = 1/\sqrt{.74} =$ $\qquad \cot\theta = 1/.53 =$

CH. 6

unit circle

distance along arc = angle

other circles

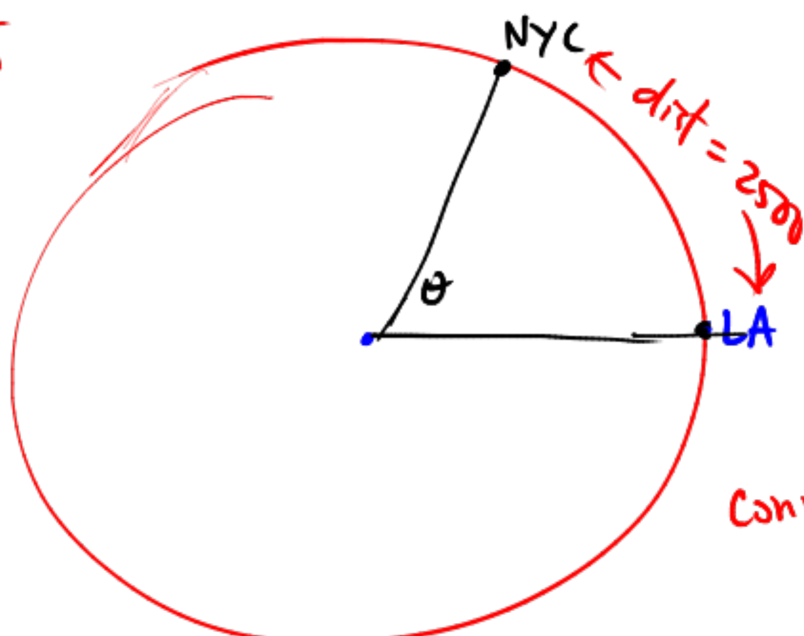


EARTH:

radius = 3960 mi

$$\boxed{S = r\theta}$$

S: length of arcs; r: radius; θ: angle



convert to deg.

$2500 = 3960\,\theta$

$$\theta = \frac{2500}{3960} = .63 \times \frac{180}{\pi} \approx 36^\circ$$