1. $\cos t \tan t = \sin t$ $\qquad 1 = \sin^2 + \cos^2$

(2) $\dfrac{\sec x}{\csc x} = \dfrac{1/\cos x}{1/\sin x} = \tan x$

(3) $\tan^2 x - \sec^2 x = \dfrac{\sin^2 x}{\cos^2 x} - \dfrac{1}{\cos^2 x} = \dfrac{-\cos^2 x}{\cos^2 x} = -1$

or $\sin^2 + \cos^2 = 1$

$\tan^2 + 1 = \sec^2$

$1 + \cot^2 = \csc$

(4) $\dfrac{\cot\theta}{\csc\theta - \sin\theta} = \dfrac{\frac{1}{\tan\theta}}{\frac{1}{\sin\theta} - \frac{\sin^2}{\sin\theta}} = \dfrac{\frac{1}{\tan\theta}}{\frac{\cos^2}{\sin\theta}} = \dfrac{1}{\tan} \cdot \dfrac{\sin}{\cos^2} = \dfrac{1}{\tan} \cdot \tan \dfrac{1}{\cos}$

$= \sec\theta$

(19) $\dfrac{1+\sin u}{\cos u} + \underbrace{\left(\dfrac{\cos u}{1+\sin u}\right)\left(\dfrac{1-\sin u}{1-\sin u}\right)}_{} =$

$\dfrac{\cos u + \cancel{\cos u \sin u}}{\cos^2 u} + \dfrac{\cos u - \cancel{\cos u \sin u}}{\cos^2 u} = \dfrac{2\cos u}{\cos^2 u} = \boxed{2\sec u}$

(11) $\dfrac{\sin x \sec x}{\tan x} = \dfrac{\sin \frac{1}{\cos x}}{\tan} = 1$

(21) $\dfrac{2 + \tan^2}{\sec^2 x} - \dfrac{\sec^2}{\sec^2 x} = \dfrac{1}{\sec^2 x} = \cos^2 x$

(23) $\tan\theta + \cos(-\theta) + \tan(-\theta)$ $\qquad \cos\theta$

ID

(29) $\dfrac{\tan y}{\csc y} = \sec y - \cos y = \dfrac{1}{\cos y} - \dfrac{\cos^2}{\cos y} = \dfrac{\sin^2 y}{\cos^2 y}$ (*)

$\dfrac{\sin y}{\cos y} \cdot \sin y = \dfrac{\sin^2 y}{\cos y}$ (*) $= \tan y \cdot \sin y$

(31) $\sin B + \cos B \cot B = \csc B$

$\dfrac{\sin^2 B}{\sin} + \dfrac{\cos^2 B}{\sin} = \dfrac{1}{\sin}$

(38) $\tan\theta + \cot\theta = \sec\theta \csc\theta = \dfrac{1}{\cos} \cdot \dfrac{1}{\sin} = \cdot$

$\dfrac{\sin^2 + \cos^2}{\cos \cdot \sin} = \dfrac{1}{\cos \cdot \sin}$

38. $\frac{\cos x}{\sec x} + \frac{\sin x}{\csc x} = 1$

40. $(\sin x + \cos x)^4 = (1 + 2\sin x \cos x)^2$

$4\sin^4 + 4\sin^3\cos + 6\sin^2\cos^2 + 4\sin\cos^3 + \cos^4 + 1 + 4\sin x\cos x + 4\sin^2\cos^2$

$(\sin x + \cos x)^2 = \sin^2 + 2\cos x\sin x + \cos^2 x$
$= 1 + 2\cos x\sin x$

47. $(1-\cos^2 x)(1+\cot^2 x) = 1$

$\sin^2 x \csc^2 = 1$

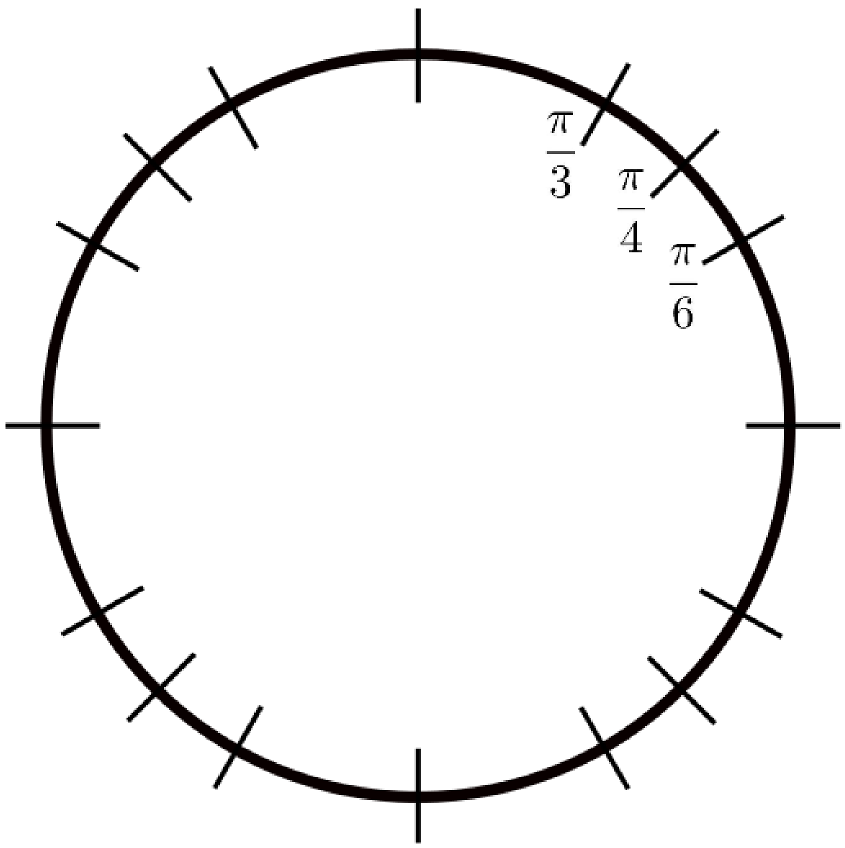
74. $\frac{1+\tan x}{1-\tan x} = \frac{\cos x + \sin x}{\cos x - \sin x} = \frac{\frac{\cos}{\cos} + \frac{\sin}{\cos}}{\frac{\cos}{\cos} - \frac{\sin}{\cos}}$

86. $\frac{\tan x + \tan y}{\frac{1}{\tan x} + \frac{1}{\tan y}} = \tan x \cdot \tan y$

$\frac{\tan x + \tan y}{\frac{\tan + \tan}{\tan \cdot \tan}}$

Name:
Trig Quiz :: Math 115
For full credit, you must circle your answers and show all your work!

1. **The unit circle with three angles is shown below.**
   a. Fill in the remaining angles as indicated by the markings.
   b. For each angle, display the corresponding terminal point (eg., $(\frac{1}{2}, \frac{\sqrt{3}}{2})$).



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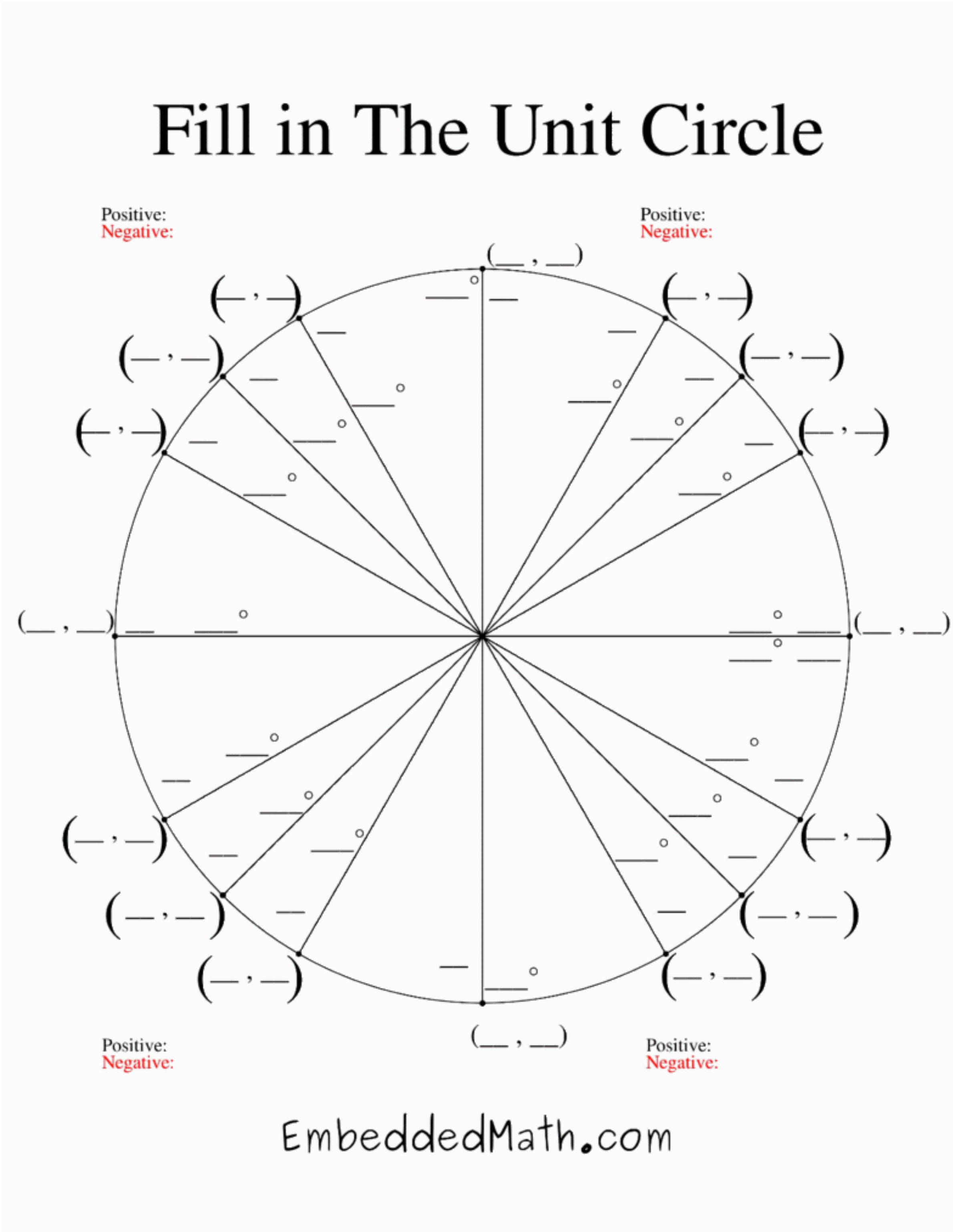
Trig Quiz :: Math 115



1. **Compute**

$\frac{5}{3} = \frac{6}{3} - \frac{1}{3}$, $\frac{6}{3} = 2$

$\sin \frac{\pi}{3}$ $\frac{\sqrt{3}}{2}$ $\cos \frac{5\pi}{3}$ $\frac{1}{2}$ $\tan 9\pi$ $\emptyset$ (0 slope)

$\frac{8\pi}{4} - \frac{\pi}{4} = \frac{7\pi}{4}$

$\sec \frac{7\pi}{4} = \frac{1}{\cos(\frac{7\pi}{4})} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

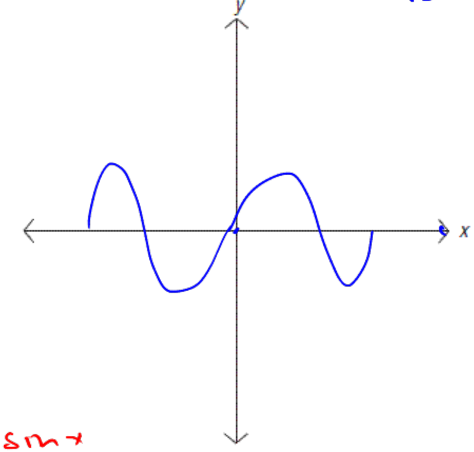
$\csc \frac{2\pi}{3} = \frac{1}{\sin\frac{2\pi}{3}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$ (same y value as $\frac{\pi}{3}$)

$\cot \frac{11\pi}{4} = \frac{1}{\tan\frac{11\pi}{4}} = \boxed{-1}$

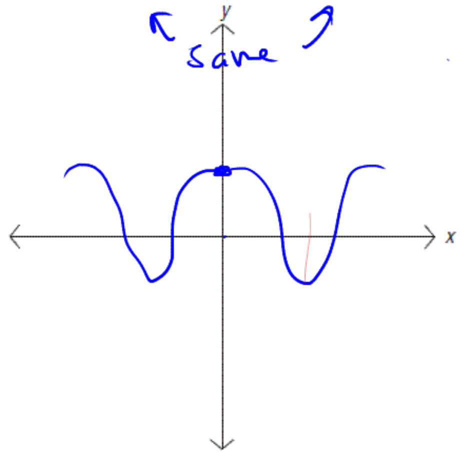
$\frac{12\pi}{4} - \frac{\pi}{4} = \frac{11\pi}{4}$, $\frac{12\pi}{4} = 3\pi$

2. **BONUS: Sketch the graphs of the functions below**

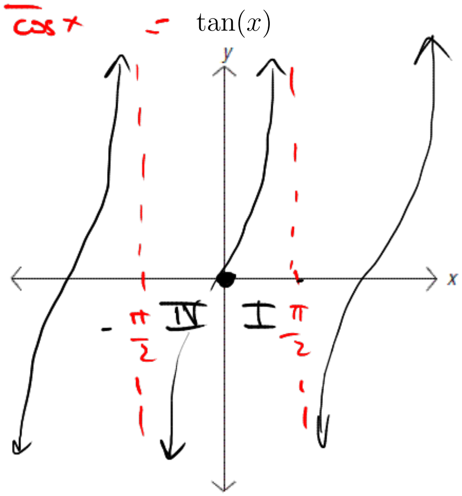
$\sin x$



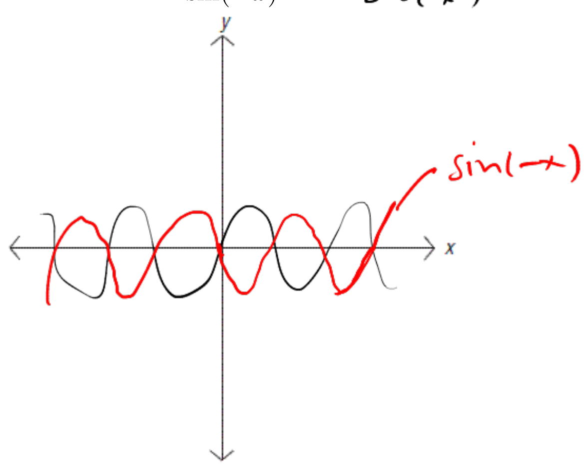
$\sin(x + \pi/2)$ AND $\cos x$ — same



$\tan(x) = \frac{\sin x}{\cos x}$



$\sin(-x) = -\sin(x)$



MA115 :: Sections 7.1 :: Trig Identities

Fundamental Trigonometric Identities

Reciprocal Identities

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1 \qquad \tan^2 x + 1 = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

true for every x-value

Here's a non-example

$\sin(1+0) = \sin(1) + \sin(0)$

Even-Odd Identities

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x \qquad \tan(-x) = -\tan x$$

guess

$\sin(x+y) = \sin(x) + \sin(y)$

$\frac{\pi}{2} \quad \frac{\pi}{2}$

$0 = 2$ ✗

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \tan\left(\frac{\pi}{2} - u\right) = \cot u \qquad \sec\left(\frac{\pi}{2} - u\right) = \csc u$$

$$\cos\left(\frac{\pi}{2} - u\right) = \sin u \qquad \cot\left(\frac{\pi}{2} - u\right) = \tan u \qquad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

Example. ~~*Simplifyig a trigonometric expression*~~ verifying the identity

$$\tan^2 x - \cancel{\sec^2 x} - 1$$

$$\frac{\sin^2 x}{\cos^2 x} - \frac{1}{\cos^2 x} = \frac{\boxed{\sin^2 x - 1}}{\cos^2 x} = \frac{-\cos^2 x}{\cos^2 x} = -1$$

$\sec^2 x$

Example. *Simplifyig by combining fractions* NO equal sign (just simplify)

$$\frac{2 + \tan^2 x}{\sec^2 x} - 1\left(\frac{\sec^2 x}{\sec^2 x}\right) = \frac{2 + \underbrace{\boxed{\tan^2 x - \sec^2 x}}_{=-1}}{\sec^2 x} = \frac{1}{\sec^2 x} = \boxed{\cos^2 x}$$

MA115 :: Sections 7.1 :: Trig Identities

Guidelines for Proving Trigonometric Identities

1. **Start with one side.** Pick one side of the equation and write it down. Your goal is to transform it into the other side. It's usually easier to start with the more complicated side.
2. **Use known identities.** Use algebra and the identities you know to change the side you started with. Bring fractional expressions to a common denominator, factor, and use the fundamental identities to simplify expressions.
3. **Convert to sines and cosines.** If you are stuck, you may find it helpful to rewrite all functions in terms of sines and cosines.

LHS

Example. *Prove an Identity by Rewriting in terms of sine and cosine*

$$\frac{\tan y}{\csc y} = \sec y - \cos y$$

$\dfrac{\frac{\sin y}{\cos y}}{\frac{1}{\sin y}} = \boxed{\dfrac{\sin^2 y}{\cos y}}$

RHS

$\dfrac{1}{\cos y} - \cos y\left(\dfrac{\cos y}{\cos y}\right) = \dfrac{1-\cos^2 y}{\cos y} = \boxed{\dfrac{\sin^2 y}{\cos y}}$

Example. *Prove an Identity by Combining Fractions*

$$\underline{\tan y \tan x} = \frac{\tan x + \tan y}{\left(\frac{\tan y}{\tan y}\right)\frac{1}{\tan x} + \frac{1}{\tan y}\left(\frac{\tan x}{\tan x}\right)} = \frac{\boxed{\tan x + \tan y}}{\frac{\tan y + \tan x}{\tan y \tan x}} = \boxed{\tan y \cdot \tan x}$$

Example. *Prove an Identity by Introducing Something Extra*

$$\frac{1+\tan x}{1-\tan x} = \frac{\cos x + \sin x}{\cos x - \sin x}$$

RHS $= \dfrac{\boxed{\frac{\cos x}{\cos x}} + \boxed{\frac{\sin x}{\cos x}}}{\boxed{\frac{\cos x}{\cos x}} - \boxed{\frac{\sin x}{\cos x}}}$

legal b/c you just multiply top & bottom by $\frac{1}{\cos x}$.

Example. *Prove an Identity by Working Both Sides Separately*

$$\tan x + \cot x = \sec x \csc x$$

LHS

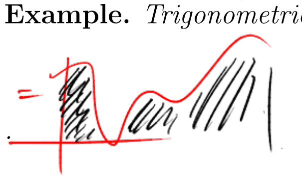
$\dfrac{\sin x}{\cos x}\dfrac{\sin x}{\sin x} + \dfrac{\cos x}{\sin x}\dfrac{\cos x}{\cos x} = \dfrac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \dfrac{1}{\sin x \cos x}$

RHS

$\dfrac{1}{\cos x} \cdot \dfrac{1}{\sin x} = \dfrac{1}{\cos x \cdot \sin x}$

Example. *Trigonometric Substitution*

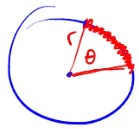
$$\frac{x}{\sqrt{1-x^2}}, \quad \boxed{x = \sin\theta}$$

$\displaystyle\int \frac{x}{\sqrt{1-x^2}}\,dx =$ 

$\dfrac{\sin\theta}{\sqrt{1-\sin^2\theta}} = \dfrac{\sin\theta}{\sqrt{\cos^2\theta}} = \dfrac{\sin\theta}{\cos\theta} = \tan\theta$

$\displaystyle\int \sec\theta \, d\theta$

6.1.8. Use



$s = r\theta$

θ = radian measure

$s = 7.5$

6.1.5 $s = r\theta$, $r = 3960$ $\theta = 4$ minutes $= \frac{4}{60}$ degrees

$= \frac{4}{60} \deg \cdot \frac{\pi}{180° \deg}$

$\theta = 4\pi/(60 \cdot 180) =$

6.2.16. $\cos\theta = -\frac{6}{12} = -\frac{1}{2}$ QIII

$\tan\theta \cot\theta = \tan\theta \cdot \frac{1}{\tan\theta} = \frac{\tan\theta}{\tan\theta} = 1$

$\csc\theta \cdot \tan\theta = \frac{1}{\cancel{\sin\theta}} \cdot \frac{\cancel{\sin\theta}}{\cos\theta} = \frac{1}{\cos\theta} = \frac{1}{-1/2} = -2$

6.2.13

$\oplus$ $\theta \in$ Quad I. $\cot\theta = 1$.

$\ominus$ $\theta \in$ Quad IV

$\cot\theta = 1$

$\frac{1}{\tan\theta} \Rightarrow \tan\theta = 1$

$\Rightarrow \theta = \pi/4$ or $5\pi/4$

$\pi/4$: this must be θ.

$5\pi/4$: quad III

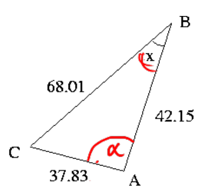
alt. sol'n: $1 + \tan^2\theta = \sec^2\theta$

$\Rightarrow 1 + 1 = \sec^2\theta$

$= 2 \Rightarrow \sec\theta = \pm\sqrt{2}$

always + in Q I & Q IV

$\Rightarrow \sec\theta = \sqrt{2} \Rightarrow \cos\theta = \frac{\sqrt{2}}{2}$



SSS $\Rightarrow$ Law of Cosines to find the largest angle.

$(68.01)^2 = (42.15)^2 + (37.83)^2 - 2(42.15)(37.83)\cos\alpha$

$4625.4 = \underbrace{1{,}776.6 + 1{,}431}_{3{,}207.7} - 3{,}189.1 \cos\alpha$

$-.4445 = \frac{1{,}417.6}{-3189.1} = \cos\alpha$

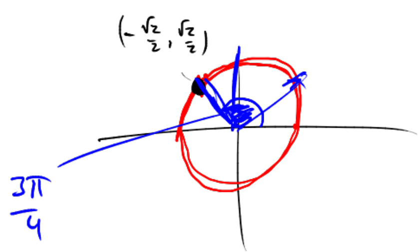
$\alpha = 2.013$ radians

or $116.39°$

$\frac{\sin(116.39)}{68.01} = \frac{\sin X}{37.83}$

$.4983 = \sin x$

$29.89° = x$



$\frac{20\pi}{4} - \frac{\pi}{4} = \frac{19\pi}{4}$

$5\pi - \frac{\pi}{4}$

MA115 :: Sections 7.1 :: Trig Identities

Fundamental Trigonometric Identities

Reciprocal Identities

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\boxed{\sin^2 x + \cos^2 x = 1} \qquad \tan^2 x + 1 = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

$$\boxed{\sin^2 x = 1 - \cos^2 x}$$

Even-Odd Identities

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x \qquad \tan(-x) = -\tan x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \tan\left(\frac{\pi}{2} - u\right) = \cot u \qquad \sec\left(\frac{\pi}{2} - u\right) = \csc u$$

$$\cos\left(\frac{\pi}{2} - u\right) = \sin u \qquad \cot\left(\frac{\pi}{2} - u\right) = \tan u \qquad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

true for every x

Example. ~~*Simplifyig a trigonometric expression*~~ Prove the identity.

$$\tan^2 x - \sec^2 x = -1$$

start: $\tan^2 x + 1 = \sec^2 x$

$-\sec^2 x - 1 \qquad -\sec^2 x - 1$

$$\tan^2 x - \sec^2 x = -1$$

Example. *Simplifyig by combining fractions*

$$\frac{2 + \tan^2 x}{\sec^2 x} - 1\left(\frac{\sec^2 x}{\sec^2 x}\right) = \frac{2 + \boxed{\tan^2 x - \sec^2 x}^{=-1}}{\sec^2 x}$$

$$= \frac{1}{\sec^2 x} = \cos^2 x$$

MA115 :: Sections 7.1 :: Trig Identities

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3. **Convert to sines and cosines.** If you are stuck, you may find it helpful to rewrite all functions in terms of sines and cosines.

Example. *Prove an Identity by Rewriting in terms of sine and cosine*

LHS RHS

$$\frac{\tan y}{\csc y} = \sec y - \cos y = \frac{1}{\cos y} - \frac{\cos^2 y}{\cos y} = \frac{1-\cos^2 y}{\cos y} = \boxed{\frac{\sin^2 y}{\cos y}}$$

$$\frac{\frac{\sin y}{\cos y}}{\frac{1}{\sin y}} = \boxed{\frac{\sin^2 y}{\cos y}}$$

Example. *Prove an Identity by Combining Fractions*

$$\frac{\tan y}{\tan y}\cdot\frac{\tan x + \tan y}{\frac{1}{\tan x} + \frac{1}{\tan y}}\left(\frac{\tan x}{\tan x}\right) = \tan x \cdot \tan y$$

$$\frac{\tan x + \tan y}{\tan y + \tan x}\cdot \tan y \cdot \tan x = \tan y \cdot \tan x$$

Example. *Prove an Identity by Introducing Something Extra*

$$\frac{1+\tan x}{1-\tan x} = \frac{\cos x + \sin x}{\cos x - \sin x}$$

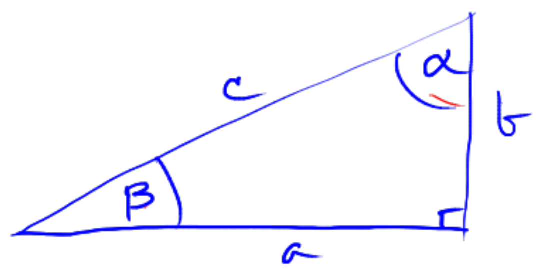
Example. *Prove an Identity by Working Both Sides Separately*

$$\tan x + \cot x = \sec x \csc x$$

Example. *Trigonometric Substitution*

$$\frac{x}{\sqrt{1-x^2}}\ ,\ x = \sin\theta$$

.



$\sin(\beta) = \frac{b}{c} = \frac{opp}{hyp}$

$\cos(\alpha) = \frac{b}{c} = \frac{adj}{opp}$

$$\begin{aligned} \alpha + \beta + 90 &= 180 \\ -90 \quad &\quad -90 \\ \hline \alpha + \beta &= 90 \\ -\beta \quad &\quad -\beta \end{aligned}$$

$\boxed{\alpha} = 90 - \beta = \frac{\pi}{2} - \beta$

substitute

$$\boxed{\sin(\beta) = \cos\left(\frac{\pi}{2} - \beta\right)}$$