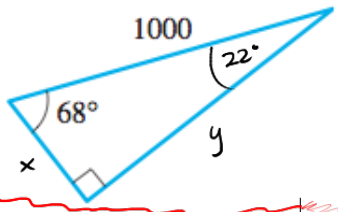
Exam 4 Study Guide :: Math 115 :: Winter 2015

1. **Solving Triangles**

$180 - 111 - 47.7 = 21.3$

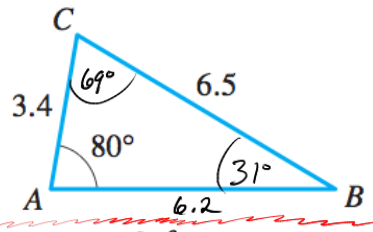


$\sin 68 \cdot 1000 = y$

$927 = y$

$\cos 68 \cdot 1000 = x$

$375 = x$



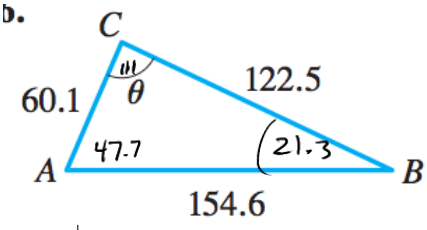
$\frac{\sin 80}{6.5} = \frac{\sin B}{3.4}$

$\Rightarrow B = \sin^{-1}\left(\frac{\sin 80}{6.5} \cdot 3.4\right) = 31°$

$\Rightarrow c = \sqrt{(3.4^2 + 6.5^2 - 2 \cdot 3.4 \cdot 6.5 \cos 69°)}$

$= 6.2$

b.



$154.6^2 = 60.1^2 + 122.5^2 - 2 \cdot 60.1 \cdot 122.5 \cdot \cos\theta$

$\Rightarrow \theta = 111°$

$\frac{\sin 111°}{154.6} = \frac{\sin A}{122.5} \Rightarrow A = 47.7°$

2. If $\sin t = 3/5$ and the terminal point of t is in quadrant II, find $\cos t, \tan t, \sec t, \cot t$ and $\csc t$.

$\left(\frac{3}{5}\right)^2 + \cos^2 t = 1$

$\frac{9}{25} + \cos^2 t = \frac{25}{25}$

$\cos^2 t = 16/25$

$\cos t = \pm 4/5$, Quad II $\Rightarrow$ choose negative for cosine

$-\frac{4}{5}$	$\frac{3/5}{-4/5} = -\frac{3}{4}$	$\frac{5}{3}$	$-\frac{4}{3}$	$\frac{5}{3}$	

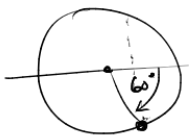
3. **Angles**
What angle in the interval $[0, 2\pi)$ is co-terminal with $17\pi/3$?

$\frac{5\pi}{3}$

$\frac{18\pi}{3} - \frac{\pi}{3} = 6\pi - \frac{\pi}{3} \Rightarrow$

$= 2\pi - \frac{\pi}{3}$

$= \frac{6\pi}{3} - \frac{\pi}{3} = \frac{5\pi}{3}$



4. **Trigonometric Equations**
Solve

$$4\cos^2\theta - 13\cos\theta + 3 = 0$$

$(4\cos\theta - 1)(\cos\theta - 3) = 0$

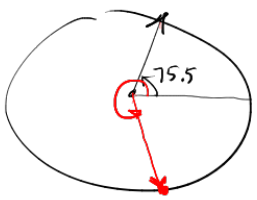
$= 0$ $\quad = 0$

$\cos\theta = 1/4$

$\theta = \cos^{-1}\left(\frac{1}{4}\right)$

$\approx 75.5°$

$\cos\theta = 3$ impossible



this θ works too! $\theta = 360 - 75.5 = 284.5$

Note: All solutions are

$75.5° + 360° \cdot n, \quad n \in \mathbb{Z}$

and

$284.5° + 360° n, \quad n \in \mathbb{Z}$

5. **Trigonometric Identities**

Verify

$\sec^4 x - \tan^4 x = \sec^2 x + \tan^2 x$

LHS: factors into

$(\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x)$

$= 1$ since $\sin^2 x + \cos^2 x = 1$ dividing by $\cos^2 x$ gives $\tan^2 x + 1 = \sec^2 x$

ALTERNATE SOLUTION

$\frac{1}{\cos^4 x} - \frac{\sin^4 x}{\cos^4 x} = \frac{1-\sin^4 x}{\cos^4 x} = \frac{(1-\sin^2 x)(1+\sin^2 x)}{\cos^4 x}$

$= \frac{1+\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$

Verify

LHS: Introduce Something New

$\frac{\cos\theta}{1-\sin\theta} = \sec\theta + \tan\theta$

$\frac{\cos\theta}{1-\sin\theta} \cdot \frac{1+\sin\theta}{1+\sin\theta} = \frac{\cos\theta + \cos\theta\sin\theta}{1-\sin^2\theta} = \frac{\cos\theta + \cos\theta\sin\theta}{\cos^2\theta} = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$

$= \sec\theta + \tan\theta$

Verify

$(\sin x - \tan x)(\cos x - \cot x) = (\cos x - 1)(\sin x - 1)$

LHS: expand

$\sin x\cos x - \sin x \cdot \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}\cdot\cos x + 1 = \sin x\cos x - \cos x - \sin x + 1$

factor out $\cos x$

notice sign change and parentheses.

$= \cos x(\sin x - 1) - (\sin x - 1)$

$= (\cos x - 1)(\sin x - 1)$ ∎

Verify

LHS: rewrite in sin/cos & combine

$\frac{\sin x + \cos x}{\sec x + \csc x} = \sin x\cos x$

$\frac{\sin x + \cos x}{\frac{1}{\cos x} + \frac{1}{\sin x}} = \frac{\sin x + \cos x}{\frac{\sin x + \cos x}{\cos x \cdot \sin x}} = \frac{\sin x + \cos x}{1} \cdot \frac{\cos x \cdot \sin x}{\sin x + \cos x} = \sin x \cdot \cos x$

Verify

LHS: $\frac{\sin^2 u}{\cos^2 u} - \frac{\sin^2 u \cdot \cos^2 u}{\cos^2 u}$

$\tan^2 u - \sin^2 u = \tan^2 u \sin^2 u$

$= \frac{\sin^2 u - \sin^2 u\cos^2 u}{\cos^2 u} = \frac{\sin^2 u(1-\cos^2 u)}{\cos^2 u} = \frac{\sin^2 u}{\cos^2 u} \cdot \frac{(1-\cos^2 u)}{1} = \tan^2 u \sin^2 u$

Verify

cosine is even

$\cos(-x) = \cos(x)$

$\cos(-x) - \sin(-x) = \cos x + \sin x$

$\cos x - (-\sin x)$

$\cos x + \sin x$

sine is odd

$\sin(-x) = -\sin(x)$

6. **Evaluations**
Evaluate exactly (no decimals)

(a) $\sin\frac{5\pi}{3}$

$\frac{5\pi}{3} = \frac{6\pi}{3} - \frac{\pi}{3} = 2\pi - \frac{\pi}{3} \leftrightarrow \frac{-\pi}{3}$ coterminal

$\sin\frac{5\pi}{3} = \sin\left(\frac{-\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

(b) $\sec\frac{6\pi}{3}$

$\sec\frac{6\pi}{3} = \sec(2\pi) = \frac{1}{\cos(2\pi)} = \frac{1}{1} = 1$

(c) $\cos\frac{5\pi}{4}$

$\frac{5\pi}{4} \in$ Quad III since $\frac{5\pi}{4} = \frac{4\pi}{4} + \frac{\pi}{4}$

$\cos\frac{5\pi}{4} = \pm\frac{\sqrt{2}}{2}$, quad III $\Rightarrow \frac{-\sqrt{2}}{2}$

$\Rightarrow$ cosine negative!

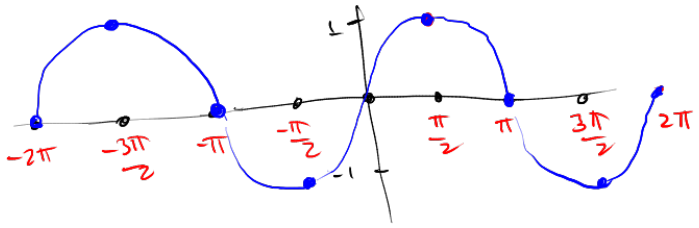
(d) $\tan\frac{17\pi}{6}$

$\tan\frac{17\pi}{6} = \frac{\sin(17\pi/6)}{\cos(17\pi/6)} = \frac{\pm\frac{1}{2}}{\pm\frac{\sqrt{3}}{2}}$ Quad III $\Rightarrow \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

7. **Graphing and Interpreting Trigonometric Functions**
Sketch a graph, determine the domain, determine all zeros, and determine the amplitude, period and phase shift (where appropriate)

(a)

$\sin x$



Domain: $\mathbb{R}$

Zeros: $n\cdot\pi$ for all $n \in \mathbb{Z}$.

Amp: 1

Period: 2π

Phase Shift: 0

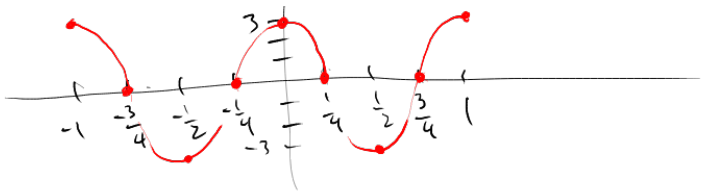
(b)

$3\cos(2\pi x)$

Amplitude: 3

Period: $\frac{2\pi}{2\pi} = 1$

Phase shift: 0

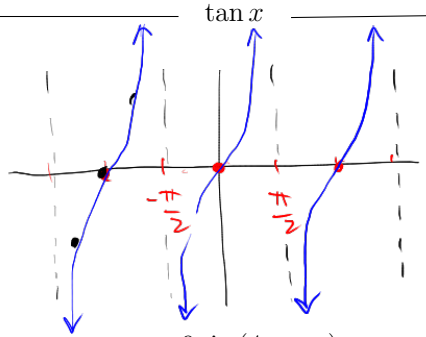


(c)

$\tan x$

x	tan x
$-\frac{\pi}{2}$	DNE
$-\frac{\pi}{4}$	-1
0	0
$\frac{\pi}{4}$	1
[illegible]	
$\frac{\pi}{2}$	DNE

Amp: N/A
Period: π
Phase shift: 0



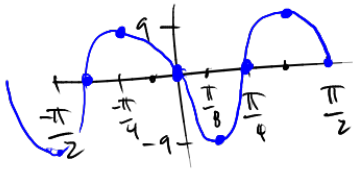
$y = 9\sin(4x - \pi)$

$y = 9\sin\left(4\left(x - \frac{\pi}{4}\right)\right)$

Amp = 9

Period: $\frac{2\pi}{4} = \frac{\pi}{2}$

Phase shift: $\frac{\pi}{4}$



8. **Trig Functions of Real Numbers**
Compute the following by hand

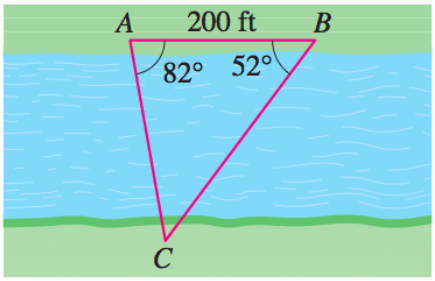
$$\sin\left(\frac{-37\pi}{6}\right) = -\sin\left(\frac{37\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$$

$$\sec\left(\frac{37\pi}{6}\right) = \frac{1}{\cos(37\pi/6)} = \frac{1}{\cos \pi/6} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

9. You and your roommate leave your room at the same time, heading to the PEIF which is 2 miles away. Your roommate walks at a speed of 4 $\frac{m}{h}$ and you ride your single speed mountain bike. Your bike has 30 inch wheels (in diameter) and your gear ratio is 2-1 (so one rotation of the pedals gives two rotations of your wheels). If you pedal an average of 40 RPMs, **(A)** what is your average speed in miles-per-hour and **(B)** how many minutes ahead of your roommate do you arrive at the PEIF?

How far does your bike travel if you roll the wheels through an angle of 45 degrees?

10. Find the shortest distance across the river as shown in the figure.



9. You and your roommate leave your room at the same time, heading to the PEIF which is 2 miles away. Your roommate walks at a speed of 4 $\frac{m}{h}$ and you ride your single speed mountain bike. Your bike has 30 inch wheels (in diameter) and your gear ratio is 2-1 (so one rotation of the pedals gives two rotations of your wheels). If you pedal an average of 40 RPMs, **(A)** what is your average speed in miles-per-hour and **(B)** how many minutes ahead of your roommate do you arrive at the PEIF?

BIKE SPEED : $80 \frac{\text{Revolution}}{\text{min}} \times \frac{30\pi \text{ inches}}{\text{Revolution}} \times \frac{1 \text{ ft}}{12 \text{ in}} \times \frac{1 \text{ mile}}{5280 \text{ ft}} \frac{60 \text{ min}}{1 \text{ hour}} = 7.14 \text{ mile/hour}$

TIME TO PEIF: $D = RT \Rightarrow T = D/R$ so

BIKE TIME: $\frac{2}{7.14} = .28 \text{ hour} \times \frac{60 \text{ min}}{1 \text{ hour}} = 16.8 \text{ min}$

WALK TIME: $\frac{2}{4} \times 60 = 30 \text{ min}$

$\Rightarrow$ (B) 13.2 minutes

How far does your bike travel if you roll the wheels through an angle of 45 degrees?

360° = 1 revolution $\leftrightarrow$ 30π inches = 7.8 feet

$45°/360° = 1/8$ th of revolution $\leftrightarrow$ $\frac{30\pi}{8}$ inches = 11.8 inches

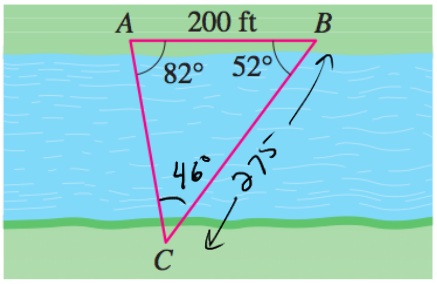
(almost 1 foot)

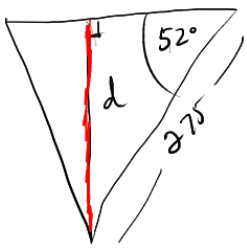
10. Find the shortest distance across the river as shown in the figure.

$C = 180 - 82 - 52 = 46$

$\frac{\sin 46}{200} = \frac{\sin 82}{BC} \Rightarrow BC = \frac{\sin 82 \cdot 200}{\sin 46} = 275$.

Now consider the right triangle where the red edge represents the shortest distance across.





$\sin 52 = \frac{d}{275}$

or

$d = \sin 52 \cdot 275$

$= 217'$