LINEAR EQUATIONS —

$$ax + b = 0$$

or

"always" get 1 solution $\quad y = mx + b \quad$ (the equation of a line)

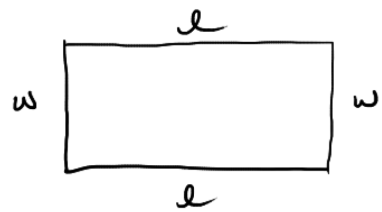
To solve ... isolate the variable.

Remember to check - ...

Ex. 200′ of fencing, plan to use it all. Our rectangular area we to enclose is $2400\ ft^2$. What are the dimensions of the garden? (length & width)

l = length
w = width

① get variables set, draw picture



② relate the varibles to given info.

Area = length × width & Perimeter = $2l + 2w$

$2400 = l \cdot w$

$200 = 2l + 2w$

$100 = l + w$

$\boxed{100 - l = w}$

③ Substitute & Solve.

$2400 = l(100 - l) = 100l - l^2$

$\boxed{l^2 - 100l + 2400 = 0}$

$(l - 60)(l - 40) = 0$

set $l - 60 = 0$ & $l - 40 \Rightarrow l = 60,\ l = 40 \Rightarrow$ $l = 60$, $w = 40$

1. $|2x-3| = 9$

this tells me immediately that

$$2x - 3 = 9 \quad \Rightarrow \quad \frac{2x}{2} = \frac{12}{2} \quad \Rightarrow \quad \boxed{x = 6}$$
$$+3 \qquad +3$$

or

$$2x - 3 = -9 \quad \Rightarrow \quad \frac{2x}{2} = \frac{-6}{2} \quad \Rightarrow \quad \boxed{x = -3}$$
$$+3 \qquad +3$$

Check: substitute $x=6$ into $|2x-3|$

$$|2(6)-3| = |12-3| = |9| = 9$$

$x = -3$ yeah

$$|2(-3)-3| = |-6-3| = |-9| = 9$$

(2) Quadratic Eqn's - $ax^2 + bx + c = 0$ a, b, c real constants

x unknown.

solutions

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

this is not magic, it's a consequence of "completing the square"

$b = 6$

$$x^2 + 6x - 1 = 0$$
$$+1 \qquad +9 \qquad +9$$

$\left(\frac{b}{2}\right)^2$

$\left(\frac{6}{2}\right)^2 = 9$

$$x^2 + 6x + 9 = 10$$
$$\sqrt{(x+3)^2} = \sqrt{10}$$
$$x + 3 = \pm\sqrt{10} \quad \Rightarrow \quad \boxed{x = -3 \pm \sqrt{10}}$$

we're given:

$$w^2 = 3(w+6)$$
$$w^2 = 3w + 18$$
$$w^2 - 3w - 18 = 0$$
$$(w-6)(w+3) = 0$$
$$w = 6 \quad \text{or} \quad w = -3$$

3. Solve for x

$$\frac{1}{y} = \frac{1}{x} + \frac{1}{z}$$

$$\frac{z}{z}\frac{1}{y} - \frac{1}{z}\frac{y}{y} = \frac{1}{x} \qquad \cancel{=} \quad y - z = x$$

$$(x)\frac{z-y}{yz} = \frac{1}{x}(x) \overset{=1}{} \quad \text{equivalent to} \quad \frac{yz}{z-y} = \frac{x}{1} = x$$

$=1$

$$\boxed{x\left(\frac{z-y}{yz}\right)\left(\frac{yz}{z-y}\right)} = 1 \cdot \left(\frac{yz}{z-y}\right)$$

$$\Rightarrow \quad \boxed{x = \frac{yz}{z-y}}$$

Quadratic Type Problems

$$(t+1)^{2} + \underbrace{8x(t+1)}_{2\cdot\sqrt{16x^2}\cdot\sqrt{(t+1)^2}} + 16x^{2} = 0$$

How to recognize the quadracticity?

Symmetry of exponents.

$$(A+B)^2 = 0 \quad \text{where} \quad \begin{aligned} A &= t+1 \\ B &= 4x \end{aligned}$$

$$\big((t+1)+4x\big)^2 = 0$$

to verify: $(t+1)^2 + 2(t+1)(4x) + (4x)^2 = 0$

$$(t+1)^2 + 8x(t+1) + 16x^2 = 0$$

More Quadratic Type Problems

exponents

$$x^{4} - 7x^{2} + 12^{0} = 0$$

$$(x^2-3)(x^2-4) = 0$$

$x^2 = 3$ $\qquad$ $x^2 = 4$

$x = \pm\sqrt{3}$ $\qquad$ $x = \pm\sqrt{4} = \pm 2$

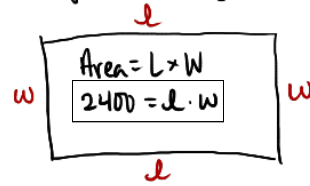
Hint! # 2 $\qquad$ Let x = amt (volume) of 60% DEET solution

what does $.60x$ represent?

1.5 Equations

- linear, easy to solve, useful:

For example — Menards get 200' of fencing. Enclose an area of 2400 ft^2.



$\boxed{200 = 2l + 2w}$

$100 - w = l$

now sub

What are the dimensions - length, width?

Step 1: Identify the unknowns, assign variables.

l = length, w = width

(what is the problem looking for?)

Step 2: Relate our variables to the (numbers) information given

$$2400 = (100 - w)w = 100w - w^2$$

rearrange

$$w^2 - 100w + 2400 = 0$$

$$(w - 60)(w - 40) = 0$$

$\Rightarrow$ $w - 60 = 0$ or $w - 40 = 0$

$w = 60$

$\boxed{w = 40 \Rightarrow l = 60}$

from $100 = l + w$

Absolute Value

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } < 0 \end{cases}.$$

Solve abs. value equations.

this $|x - 1| = 5$

CHECK

sub $x = 6$

$|6 - 1| = 5$

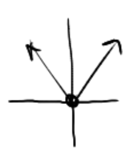
$|-4 - 1| = |-5| = -(-5) = 5$

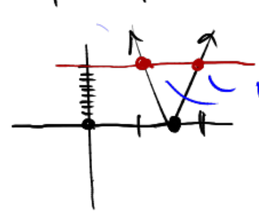
means $x - 1 = 5 \Rightarrow x = 6$

$x - 1 = -5 \Rightarrow x = -4$

Solve $|2x - 3| = 9 \Rightarrow$ $2x - 3 = 9 \Rightarrow 2x = 12, x = 6$

$2x - 3 = -9$, $2x = -6, x = -3$





we're looking for the x-coords of these points.

Check: $|12 - 3| = 9$

$|2(-3) - 3| = |-6 - 3| = 9$

3. $\frac{1}{y} = \frac{1}{x} + \frac{1}{z}$ Solve for x. (Isolate x)

$-\frac{1}{z}$ $-\frac{1}{z}$

$\underbrace{\frac{z}{z}\frac{1}{y} - \frac{1}{z}\frac{y}{y}} = \frac{1}{x}$ $\neq$ $y - z = x$

$\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ not same as $2 - 4 = 4$

$\frac{z-y}{zy} = \frac{1}{x}$ this means $x = \frac{zy}{z-y}$

cross multiply.

#2. $w^2 = 3(w+6)$ quadratic type highest degree is 2.

$w^2 = 3w + 18$

$w^2 - 3w - 18 = 0$

$(w-6)(w+3) = 0$

$w = 6, \; w = -3$

Also $(t+1)^2 + 8x(t+1) + 16x^2 = 0$ factor

looks like $\underbrace{A^2 + 2AB + B^2}_{(A+B)^2} = 0$

$((t+1) + 4x)^2 = 0$

$A = (t+1)$

$B = 4x$

Also $\quad x^4 - 7x^2 + 12 = 0$

$A = x^2$
substitute...

$A^2 - 7A + 12 = 0$

$(A-3)(A-4) = 0$

$A = 3\ ,\ A = 4$

substitute back

$x^2 = 3\ ,\ x^2 = 4$

$x = \pm\sqrt{3}\ ,\ x = \pm\sqrt{4} = \pm 2$

Two other methods for solving quadratics

given $\quad ax^2 + bx + c = 0 \quad$ where a, b, c are real

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

other method:
complete the □

$x^2 - 6x + 1 = 0$
$\quad\quad\quad\ -1 \quad -1$

$x^2 - 6x = -1$

$\left(\frac{-6}{2}\right)^2 = 9$

$\underbrace{x^2 - 6x + 9} = +9 - 1$

$(x-3)^2 = 8$

$x - 3 = \pm\sqrt{8} = \pm 2\sqrt{2}$

$$\boxed{x = 3 \pm 2\sqrt{2}}$$