Name: **MA115 :: Exam 1 Practice**

For full credit, circle your answers and show all your work!

1. Solve the inequality $3|2x+1| - 5 < 12$

$3|2x+1| < 17$

$|2x+1| < \frac{17}{3}$

$-\frac{17}{3} < 2x+1 < \frac{17}{3}$

$-1 \quad -1$

$-\frac{3}{3} \quad -\frac{3}{3}$

$-\frac{20}{3} < 2x < \frac{14}{3}$

$-\frac{10}{3} < x < \frac{7}{3}$

$\left(-\frac{10}{3}, \frac{7}{3}\right)$

Simplify

2. ~~Factor~~ the expressions below:

(a)

$$\frac{\sqrt{\frac{(b+y)^3}{(a+x)}}}{b+y} = \frac{\left(\frac{(b+y)^3}{(a+x)}\right)^{1/2}}{b+y} = \frac{\frac{(b+y)^{3/2}}{(a+x)^{1/2}}}{b+y} = \frac{(b+y)^{3/2}}{(a+x)^{1/2}} \cdot \frac{(b+y)^{-1}}{1} = \frac{(b+y)^{1/2}}{(a+x)^{1/2}} = \left(\frac{b+y}{a+x}\right)^{1/2}$$

$\frac{3}{2} - \frac{2}{2} = \frac{1}{2}$

(b)

$$\frac{(A+B)^2 - 2AB}{A^2+B^2} = \frac{A^2+2AB+B^2 - 2AB}{A^2+B^2} = \frac{A^2+B^2}{A^2+B^2} = 1$$

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3. Simplify the expression and eliminate any negative exponents:

1. remove parenthesis / distribute
2. simplify on like levels

$$\frac{b^{-1}(bd)^2c}{(ab^{-1}d)^2a^{-2}ba^{-1}b} = \frac{b^{-1}b^2d^2c}{d^2b^{-2}d^2a^{-2}ba^{-1}b}$$

$$= \frac{bd^2c}{d^2a^{-1}} = abc$$

4. Rationalize the Numerator:

In calculus you will also see

$$\frac{\sqrt{a+h}-\sqrt{a}}{h}\cdot\frac{\sqrt{a+h}+\sqrt{a}}{\sqrt{a+h}+\sqrt{a}}$$

$$= \frac{(a+h)-a}{h(\sqrt{a+h}+\sqrt{a})} = \frac{h}{h(\sqrt{a+h}+\sqrt{a})}$$

$$= \frac{1}{\sqrt{a+h}+\sqrt{a}}$$

$$\frac{\sqrt{a+h}-a}{h}\cdot\frac{\sqrt{a+h}+a}{\sqrt{a+h}+a} = \frac{(a+h)-a^2}{h[\sqrt{a+h}+a]}$$

$= 1$

opp signs $\Rightarrow$ inner stuff cancel

$=$

5. Factor the expression completely and simplify your answer. Write your answer with positive exponents. Begin by factoring out the lowest power of each common factor.

$$(x^2+3)^{-1/3} - x^2(x^2+3)^{-4/3}$$

$-1/3 - (-4/3)$

$-1/3 + 4/3 = 3/3 = 1$

1. Lowest Power of common factor $(x^2+3)^{-4/3}$
2. Factor out
3. "Subtract exponents"

$$(x^2+3)^{-4/3}\left((x^2+3) - x^2(x^2+3)^{1}\right)$$

$-4/3 - (-4/3) = 0$ ← exp. that was there; exp you removed

$$(x^2+3)^{-4/3}\left[\frac{(x^2+3) - x^2}{3}\right]$$

$$\boxed{3(x^2+3)^{-4/3}}$$

6. Perform the indicated operations and simplify:

$$\frac{(x+h)^{-2} - x^{-2}}{h}$$

(1) $\dfrac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$ $\quad \frac{x^2}{x^2}\cdot\frac{1}{(x+h)^2}$, $\frac{(x+h)^2}{(x+h)^2}$

(2) $\dfrac{\frac{x^2-(x+h)^2}{x^2(x+h)^2}}{h}$ $\longrightarrow x^2-(x^2+2xh+h^2)$

(3) $\dfrac{\frac{x^2-x^2-2xh-h^2}{x^2(x+h)^2}}{h} = \dfrac{-2xh-h^2}{x^2(x+h)^2}\cdot\dfrac{1}{h} = \dfrac{-h(2x+h)}{h\,x^2(x+h)^2} = \boxed{\dfrac{-(2x+h)}{x^2(x+h)^2}}$

7. Solve the inequality: non-linear

$$x < -\frac{21}{x-10}$$

1. Don't cross multiply
2. Use +/− to gather into a super-fraction

$$\left(\frac{x-10}{x-10}\right)x + \frac{21}{x-10} < 0$$

3. $$\frac{x^2 - 10x + 21}{x-10} < 0$$

4. $$\frac{(x-7)(x-3)}{x-10} < 0$$

$\frac{x^2}{x} = x$

$$\frac{(x-7)^2(x-3)}{x-10}$$

Zeros

$(x-7)(x-3) = 0$

$x-7=0$, $x=7$

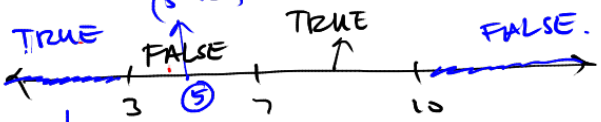
$x-3=0$, $x=3$

Breaks

$x-10=0$

$x=10$

$$\frac{(5-7)(5-3)}{(5-10)} > 0$$



choose pts away from these to test

8. Factor the expression completely.

4 terms ... think grouping.

$x^5 + 6x^3 + x^2 + 6$

$x^3(x^2+6) + (x^2+6)$

$(x^2+6)(x^3+1)$

9. Find all solutions to the equations:

$(x^2+4x)^2 + 2(x^2+4x) - 3 = 0$

Basics

$A^2 + 2A - 3 = 0$

$(A+3)(A-1) = 0$

$\sqrt{20} = \sqrt{4\cdot 5} = 2\sqrt{5}$

$((x^2+4x)+3)((x^2+4x)-1) = 0$

two terms multiplied give 0 means one of the terms must be 0

$(x^2+4x)+3 = 0$

$(x^2+4x+3) = 0$

$(x+3)(x+1) = 0$

$x = -3, -1$

$(x^2+4x)-1 = 0$

$x^2+4x-1 = 0$

$x = \frac{-4 \pm \sqrt{16 - 4\cdot 1\cdot(-1)}}{2}$

$= \frac{-4 \pm \sqrt{20}}{2} = -2 \pm \sqrt{5}$

$\sqrt{20} = 2\sqrt{5}$

4 answers

10. The line segment between $(2, 1)$ and $(-5, 2)$ is called AB. Find the equation of the line that intersects AB at its midpoint and is perpendicular to AB.

Look below on second copy.

11. A wizard wants to make a Polyjuice Potion that is exactly 8% fluxweed. He will do so by mixing Terrible Tea, which is 10.5% fluxweed and Tasty Tonic which is 4.3% fluxweed. How much of each should the wizard use in order to create 10 oz of Polyjuice Potion.

FLUXWEED: Total Fluxweed = Tea Fluxweed + Tonic Fluxweed

10 oz @ 8% = (x) @ 10.5% + (10 − x) @ 4.3%

x = amt of Tea

$(.08)10 = .105x + (10-x)(.043)$

1 equation
1 unknown
=> you can solve.

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For full credit, circle your answers and show all your work!

step 1: $|blah| < 0$

1. Solve the inequality $3|2x+1|-5<12$

$+5 \quad +5$

$\frac{3|2x+1|}{3} < \frac{17}{3}$

$|2x+1| < \frac{17}{3}$

$-\frac{3}{3} \quad -\frac{17}{3} < 2x+1 < \frac{17}{3} \quad \frac{3}{3}$

-1

Simplify

$\frac{1}{2}\cdot\frac{-20}{3} < \frac{2x}{2} < \frac{14}{3}\cdot\frac{1}{2}$

$-\frac{10}{3} < x < \frac{7}{3}$

$\left(-\frac{10}{3}, \frac{7}{3}\right)$

2. Factor the expressions below:

(a)

$$\frac{\left(\frac{(b+y)^3}{a+x}\right)^{1/2}}{b+y} = \frac{\left((b+y)^3\right)^{1/2}}{(a+x)^{1/2}} = \frac{\sqrt{\frac{(b+y)^3}{(a+x)}}}{b+y}$$

$\frac{3}{2}-1 = \frac{3}{2}-\frac{2}{2} = \frac{1}{2}$

$$= \frac{(b+y)^{3/2}}{(a+x)^{1/2}} \cdot \frac{(b+y)^{-1}}{1} = \frac{(b+y)^{\frac{1}{2}}}{(a+x)^{\frac{1}{2}}} = \sqrt{\frac{b+y}{a+x}}$$

(b)

$$\frac{(A+B)^2-2AB}{A^2+B^2}$$

cancel

$$\frac{(A^2+2AB+B^2)-2AB}{A^2+B^2} = \frac{A^2+B^2}{A^2+B^2} = 1$$

3. Simplify the expression and eliminate any negative exponents:

1. remove parenthesis distribute
2. work level by level

$$\frac{b^{-1}(bd)^2c}{(ab^{-1}d)^2a^{-2}ba^{-1}b} = \frac{b^{-1}b^2d^2c}{a^2b^{-2}d^2a^{-2}ba^{-1}b} = \frac{bd^2c}{d^2a^{-1}} = abc$$

4. Rationalize the Numerator:

Commonly in Calc.

$$\frac{\sqrt{a+h}-\sqrt{a}}{h}\left(\frac{\sqrt{a+h}+\sqrt{a}}{\sqrt{a+h}+\sqrt{a}}\right)$$

$$\frac{(a+h)-a}{h(\sqrt{a+h}+\sqrt{a})} = \frac{h}{h(\sqrt{a+h}+\sqrt{a})} = \frac{1}{\sqrt{a+h}+\sqrt{a}}$$

$$\frac{\sqrt{a+h}-a}{h}\left(\frac{\sqrt{a+h}+a}{\sqrt{a+h}+a}\right)$$

$$\frac{(a+h)+a\sqrt{a+h}-a\sqrt{a+h}-a^2}{h(\sqrt{a+h}+a)} = \frac{a+h-a^2}{h(\sqrt{a+h}+a)}$$

Multiply like Bases: add exponents

Factor Out Like Bases: subtract exponents

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5. Factor the expression completely and simplify your answer. Write your answer with positive exponents. (Begin by factoring out the lowest power of each common factor.)

$(x^2+3)^{-1/3} - x^2(x^2+3)^{-4/3}$ → $-\frac{1}{3} - \left(-\frac{4}{3}\right) = \frac{3}{3} = 1$

$(x^2+3)^{-4/3}\left((x^2+3) - x^2(x^2+1)\right)$

$(x^2+3)^{-4/3}\left(x^2+3 - x^2\right)$

$3(x^2+3)^{-4/3} = \dfrac{3}{(x^2+3)^{4/3}}$

6. Perform the indicated operations and simplify:

$$\frac{(x+h)^{-2} - x^{-2}}{h}$$

flip & square

$$\frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} \cdot \frac{x^2(x+h)^2}{x^2(x+h)^2}$$

$$= \frac{x^2 - (x+h)^2}{x^2(x+h)^2} \Big/ h$$

$x^2 + 2xh + h^2$

$$= \frac{x^2 - x^2 - 2xh - h^2}{x^2(x+h)^2} \Big/ h$$

$0 \qquad -h(2x+h)$

$$= \frac{-h(2x+h)}{x^2(x+h)^2} \cdot \frac{1}{h}$$

$$= -\frac{(2x+h)}{x^2(x+h)^2}$$

non-linear

7. Solve the inequality:

$$x < -\frac{21}{x-10}$$

1. do not cross multiply.
2. use +/− to get Fraction < 0
3. Find zeros of fraction & breaks

$$\left(\frac{x-10}{x-10}\right)x + \frac{21}{x-10} < 0$$

$$\frac{x^2-10x+21}{x-10} < 0$$

ZEROS: $x^2-10x+21=0$, $(x-7)(x-3)=0$, $x=7, x=3$

BREAKS: $x=10$

For large x: $\approx \frac{x\cdot x}{x} = x$

$\frac{(x-7)(x-3)}{(x-10)} < 0$

$\frac{(5-7)(5-3)}{(5-10)} = \frac{(-)(+)}{(-)} > 0$

$\frac{(x-7)^2(x-3)}{(x-10)}$

$(-\infty, 3) \cup (7, 10)$

8. Factor the expression completely.

4 terms, think groupings

$x^5 + 6x^3 + x^2 + 6$

$x^3(x^2 + 6) + (x^2 + 6)$

x^2+6 $\quad (x^3 + 1)$

$(x^2+6)(x^3+1)$

9. Find all solutions to the equations:

Think:

$A^2 + 2A - 3 = 0$

$(A+3)(A-1) = 0$

$(\overbrace{x^2+4x}^{A})^2 + 2(\overbrace{x^2+4x}^{A}) - 3 = 0$

$\big((x^2+4x) + 3\big)\big((x^2+4x) - 1\big) = 0$

$= 0$ $\qquad = 0$

$x^2+4x+3=0$

$(x+3)(x+1)=0$

$x = -3, -1$

$x^2+4x-1=0$

$x = \dfrac{-4 \pm \sqrt{16 - 4\cdot 1(-1)}}{2}$

$= \dfrac{-4 \pm \sqrt{20}}{2}$

$= \dfrac{-4 \pm \sqrt{4}\sqrt{5}}{2} = \dfrac{-4 \pm 2\sqrt{5}}{2} = \dfrac{1}{2}(-4 \pm 2\sqrt{5})$

$= -2 \pm \sqrt{5}$

4 sols

$\dfrac{A\sqrt{5}}{A} = \sqrt{5}$

$\dfrac{2\sqrt{5}}{2}$

10. The line segment between $(2, 1)$ and $(-5, 2)$ is called AB. Find the equation of the line that intersects AB at its midpoint and is perpendicular to AB.

midpt = $\left(\frac{2-5}{2}, \frac{1+2}{2}\right) = \left(-\frac{3}{2}, \frac{3}{2}\right)$

slope = $\frac{2-1}{-5-2} = \frac{1}{-7} \Rightarrow m_\perp = 7$

$y - \frac{3}{2} = 7\left(x + \frac{3}{2}\right) = 7x + \frac{21}{2}$

so $y = 7x + \frac{24}{2} \Rightarrow$ $y = 7x + 12$

11. A wizard wants to make a Polyjuice Potion that is exactly 8% fluxweed. He will do so by mixing Terrible Tea, which is 10.5% fluxweed and Tasty Tonic which is 4.3% fluxweed. How much of each should the wizard use in order to create 10 oz of Polyjuice Potion.

FluxWeed: Amt @ End = Amt in Tea + Amt in Tonic

8% of 10 oz $= 10.5\%$ of $(x) + 4.3\%\,(10-x)$

x = Amt. of Tea

1 eqn. 1 variable $\Rightarrow$ linear.