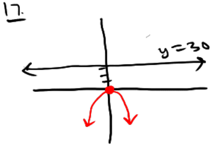
THURSDAY - Jan 29 - Text: 1.10 & 1.11 —— Lines & Variation

17.



WeBWorK: $\cdot\ -2x^2 \le 30$

$y = 30$

$y = -2x^2 + 0$

$-\frac{1}{2}\left(-2x^2 - 30 \le 0\right)$

(*) $\boxed{x^2 + 15 \ge 0}$.

$x^2 \ge -15$

If $x = -1$
$\Rightarrow x^2 + 15 = 16 > 0$
$x = +1$
$x^2 + 15 = 16 > 0$

Here, notice $x^2 \ge 0$ so $x^2 + 15$ is always > 0

sol'n $(-\infty, \infty)$

#21 $\frac{x}{x-1} > \frac{x}{7}$

• Push everything into 1 super-fraction on left, 0 on right

$\frac{x}{x-1}\left(\frac{7}{7}\right) - \frac{x}{7}\left(\frac{x-1}{x-1}\right) > 0$

$\boxed{\frac{7x - x^2 + x}{7(x-1)} > 0}$

zeros
$7x - x^2 + x = 0$
$-x^2 + 8x = 0$
$-x(x-8) = 0$
$x = 0, x = 8$

breaks
$7(x-1) = 0$
$x = 1$

test factored form

(*) $\frac{-x(x-8)}{7(x-1)} > 0$

$\frac{\ominus\ominus}{7(+)} > 0$

true | 0 | FALSE | 1 | 2 TRUE | 8 | False

test

$\frac{-x \cdot x}{7 \cdot x} = \frac{-x}{7}$

$\Rightarrow < 0$ when $x \gg 8$.

pick $x \in (0,1)$
$x = .5$

$\frac{-.5(.5-8)}{7(.5-1)} = \frac{\ominus\ominus}{7\ominus} < 0$

plug into (*)

$\frac{-2(2-8)}{7(2-1)} \overset{?}{>} 0$

$\frac{-2(-6)}{7(1)} = \frac{12}{7} > 0$

$\boxed{(-\infty, 0) \cup (1, 8)}$

For large x
$x - 8 \approx x$
$x - 1 \approx x$

$\frac{-x(x)}{7(x)} = \frac{-x}{7}$ ⟵ at $x = -1000$ this > 0

8 $\frac{1}{R} = \frac{R_2}{R_2}\frac{1}{R_1} + \frac{1}{R_2}\frac{R_1}{R_1}$ Solve for R_1

$\frac{1}{R} = \frac{R_2 + R_1}{R_1 R_2}$ cross mult

$R(R_2 + R_1) = R_1 R_2$

$R R_2 + R R_1 = R_1 R_2$
$\quad - R R_1 \quad - R R_1$

$R R_2 = R_1 R_2 - R R_1$
$= R_1(R_2 - R)$

$\frac{R R_2}{R_2 - R} = R_1$

3. $\left|\frac{7x+5}{8}\right| \le 1$

$-1 \le \frac{7x+5}{8} \le 1$

$-8 \le 7x + 5 \le 8$
$-13 \le 7x \le 3$
$-\frac{13}{7} \le x \le \frac{3}{7}$ $\Rightarrow \left[-\frac{13}{7}, \frac{3}{7}\right]$

(18) Two plans: (A) \$30 + 12 cent/mile
(B) \$50 unlimited miles

for what range of miles is (B) cheaper?

x = # miles $\$50 < \$30 + .12x$

$20 < .12x$

(166) $= \frac{500}{3}\ \frac{1000}{6} = \frac{2000}{12}$ $= \frac{20}{.12} = x$

#20

$F = \frac{9}{5}C + 32$

Inclusive ($\leq$)

$\Rightarrow$ $-6 \leq F \leq 33$

$-6 \leq \frac{9}{5}C + 32 \leq 33$

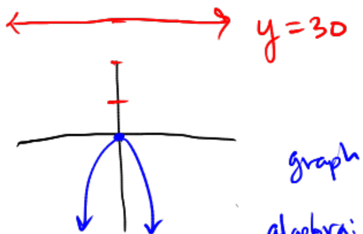
$-38 \leq \frac{9}{5}C \leq 1$ ok in ww.

$\left[\frac{5}{9}\cdot(-38),\ \frac{5}{9}\right]$

17. $-2x^2 \leq 30$

unhappy ∩ (b/c -2)

Solution = set of x-values for which graph $y = -2x^2$ is below graph $y = 30$



graph tells you $-2x^2$ is always ≤ 30.

$(-\infty, \infty)$

algebra: $-2x^2 \leq 30$

$x^2 \geq -15$

$x^2 + 15 \geq 0$

doesn't factor so just test any point since $x^2 + 15 \neq 0$.

test $x = 0 \Rightarrow x^2 + 15 \geq 0$

$\frac{1}{R} = \frac{R_2}{R_2}\frac{1}{R_1} + \frac{1}{R_2}\frac{R_1}{R_1}$ Solve for R_1

$\frac{1}{R} = \frac{R_2 + R_1}{R_2 R_1}$

cross mult.

–distribut– remove parenthesis

$R_2 R_1 = R(R_2 + R_1) = RR_2 + RR_1$

gather R_1 terms

$R_2 R_1 - RR_1 = RR_2$

factor

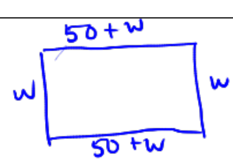
$R_1(R_2 - R) = RR_2$

$R_1 = RR_2 / (R_2 - R)$

(10)

Rectangle is (length) 50cm longer than it is wide. Express the perimeter of this rectangle in terms of the width.

l = length, $l = 50 + w$



$P = 2l + 2w = 2(50 + w) + 2w = 4w + 100$

LINES & VARIATION - 1.10 & 1.11 in your text.

VARIOUS EQN'S OF LINES

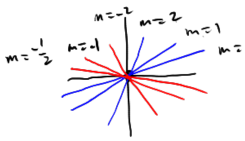
$y = mx + b$ — slope-intercept form (m = slope, b = intercept)

$y - y_1 = m(x - x_1)$ — point-slope formula (m = slope; (x_1, y_1) → lives on line.)

$ax + by = c$ — general equation

$m = (y - y_1)/(x - x_1)$ — slope formula

(solve for m: the margin note links the point-slope formula to the slope formula)



$m = \frac{1}{2}$, $m = 1$, $m = 2$ (blue)
$m = -\frac{1}{2}$, $m = -1$, $m = -2$

horizontal line: ⟵⟶ slope = 0, $y = 5$ eg. $y = \pi$

vertical line ↕ slope = und (has infinite slope)

parallel = // same slope

perpendicular, intersect at 90, slope = $-\frac{1}{m}$

slope = $\frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$ = rate of change (Δy → change in y)

VARIATION: x is proportional to y

$x = k \cdot y$ $k \in \mathbb{R}$, unknown, depends on context.

x is inversely proportional to y

$x = k \cdot \frac{1}{y} = \frac{k}{y}$

x is jointly proportional to y & w

$x = kyw$

x varies directly as the cube of y. $x = ky^3$
w is jointly proportional to the square of x and the cube root of y and inversely proportional to z $w = \frac{(kx^2)\sqrt[3]{y}}{z}$

We are 20 feet from fire. (Wood is then doubled.) How far from the fire must we move to feel the same heat as before? ↳ we double w

h = heat we feel
w = amt. of wood
d = dist.
= 20 (initially)

$h = \frac{kw}{d^3}$

$h = \frac{kw}{20^3}$ (heat we feel at 20')

$\frac{kw}{20^3} = \frac{k(2w)}{d_1^3}$ (after adding wood)

↑ equal to ensure the same heat is felt.

cross mult.

$\sqrt[3]{d_1^3} = \sqrt[3]{20^3 \cdot 2} =$

$d_1 = 20 \cdot \sqrt[3]{2} \approx 22'$ new dist.

Lines: slope $= \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$ = rate of change

$y = mx + b$ slope-intercept
m b

$y - y_1 = m(x - x_1)$ point-slope
(x_1, y_1) m
↳ lives on line

solve for m

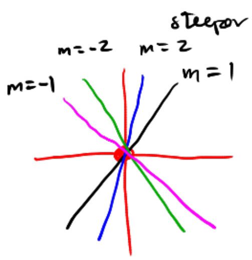
slope formula $m = \frac{y - y_1}{x - x_1}$

Parallel
same slope

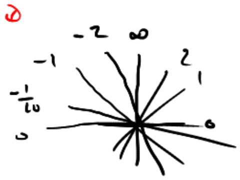
Perpendicular
90° intersect
$-\frac{1}{m}$

Horizontal
⟷
$y = \pi$
slope = 0

Vertical
↕
$x = 1$
slope of vertical line: undefined "infinite"

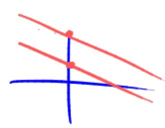


$m = 1$
$m = 2$
$m = -1$
$m = -2$



Consider these lines. Are they //? yea

$x + y = 1 \Rightarrow y = -x + 1$
$x + y = 5$ $y = -x + 5$



Variation: "x is proportional to y" aka varies directly
this means in math:

$x = ky$ for some unknown $k \in \mathbb{R}$ which depends on context

x "varies indirectly (inversely)" with y
means

$x = k\frac{1}{y}$

x varies jointly with w & y

$x = kwy$

Ex.

The amount of heat (h) you feel standing by a campfire is proportional to the amount of wood (w) and on the fire and inversely prop. to the cube of the distance (d) to the fire.b

$h = \frac{k \cdot w}{d^3}$