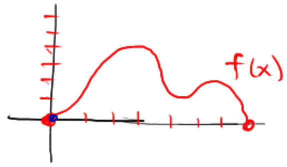
2.2 & 2.3 Graphs & Rates of Change of Functions

The graph is the set of points $\{(x, f(x)) \mid x \in \text{Domain}(f)\}$

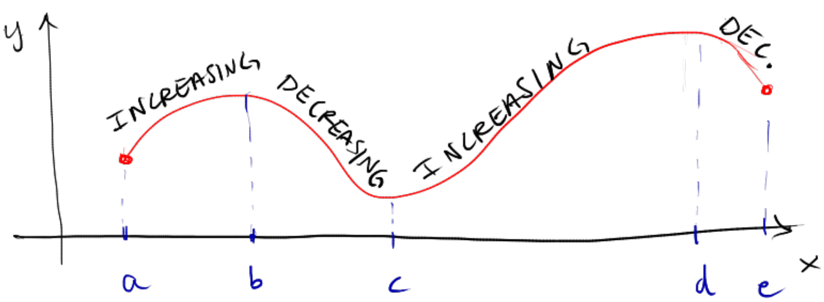
For the graph of f(x) shown ...



... we learn that $f(3)=4$
$f(0)=0$
$f(6)=2$
$f(7)=0$

We also infer from the graph that $\text{domain}(f) = [0,7]$.

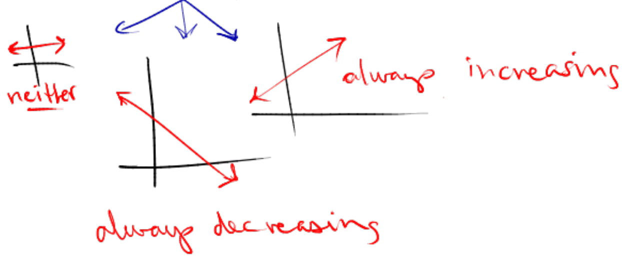
Functions describe changing quantities (miles, pounds, heights, money)
– it's very important to be able to identify periods when the quantities are increasing or decreasing.



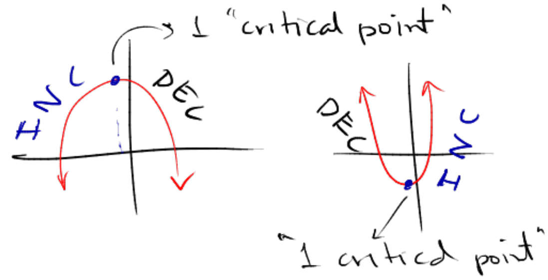
$y = f(x)$

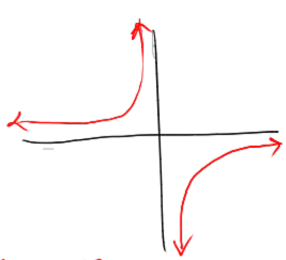
f is increasing on $(a,b) \cup (c,d)$ i.e., $a<b \Rightarrow f(a)<f(b)$
decreasing on $(b,c) \cup (d,e)$ $b<c \Rightarrow f(b)>f(c)$

Ex.: Lines,



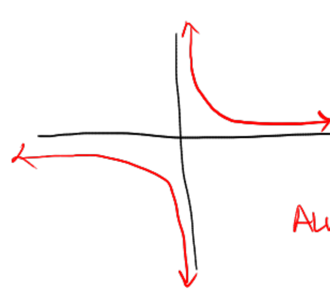
Parabolas





$f(x) = 1/x$

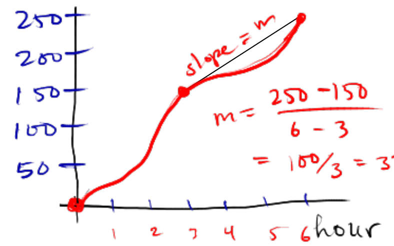
ALWAYS INCREASING



$f(x) = -1/x$

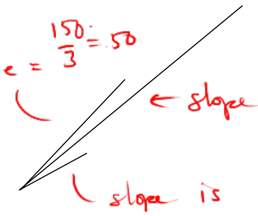
ALWAYS DECREASING

AVERAGE RATE OF CHANGE



$m = \frac{250-150}{6-3}$

$= 100/3 = 33 \frac{m}{h}$



slope = $\frac{150}{3} = 50$

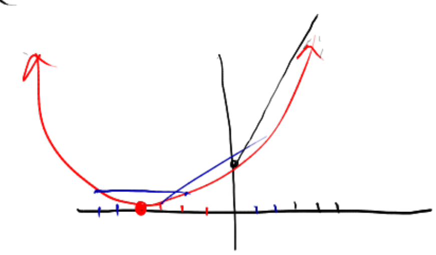
← slope is $\frac{\text{rise}}{\text{run}} = \frac{250}{6} \approx 42$

slope is $\frac{50}{2} = 25$

FORMULA: $\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$ for $a<b$ = slope of secant line between $(a,f(a))$ & $(b,f(b))$

Example: $f(x) = (x+4)^2$ calculate avg. rate of change between:

		AROC	
(a)	$x=-3$, $x=2$	7	$\frac{f(-3)-f(2)}{-3-2} = \frac{1-36}{-5} = 7$
(b)	$x=-6$, $x=-2$	0	
(c)	$x=0$, $x=5$	13	$\frac{f(5)-f(0)}{5-0} = \frac{81-16}{5} = \frac{65}{5} = 13$
(d)	$x=5$, $x=5.1$	small change late is big	$\frac{f(5.1)-f(5)}{5.1-5} = \frac{9.1^2-81}{.1} = \frac{1.81}{.1} = 18$
(e)	$x=-4.1$, $x=-3.8$	.1 small change early near vertex is small.	$\frac{f(-3.8)-f(-4.1)}{-3.8-(-4.1)} = \frac{(.2)^2-(.1)^2}{.3}$



$= \frac{3/100}{3/10} = 10/100 = \frac{1}{10}$

$= .1$

Ex. Falling objects:

$f(t) = -16t^2 + 100$ Height of object dropped from 100' high — t seconds after release

$f(t) = 0 = -16t^2 + 100$

$100/16 = t^2 \Rightarrow t = 10/4 = 2.5$ seconds till impact.

About how fast is it going on impact?

Initially, near release — $\frac{f(.5)-f(.4)}{.5-.4} = -14.4$ ft/sec

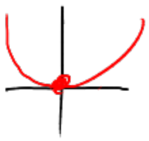
$\frac{f(2.5)-f(2.4)}{2.5-2.4} = -78.4$ ft/sec

Homework Sheets:

warm-up hint: what's the slope of the radial line?

back side:

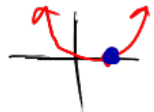
X^2



x^2+1

x^2-1

$(x-1)^2$ right



→ plug in $x=1$

$(x+1)^2$ left