③ $f(x) = 3 + 4x^2 - x^4$

Max value:

$w = x^2 \Rightarrow$

maximize:

$3 + 4w - w^2$

occurs at $\left(\frac{-b}{2a}\right)$ $w = \frac{-4}{2(-1)} = 2$

or $x^2 = 2$

so $x = \pm\sqrt{2}$

$-(x^4 - 4x^2 + 4) + 7 = -(x^2-2) + 7$

$\Rightarrow x^2 = 2$

$x = \pm\sqrt{2}$

$= -(w^2 - 4w + 4) + 3 + 4$

$= -(w-2)^2 + 7$

occurs when $w = 2$

x^2 so $x = \pm\sqrt{2}$

Check: $f(\pm\sqrt{2}) = 3 + 4(2) - 4 = 7$

vs $f(1) = 3 + 4 - 1 = 6$

$f(3) = 3 + 4(9) - 81 = 3 + 36 - 81$ (not large)

even even
$x^2 + x^4$ } even
$x^6 + x^2$ ...

$f(x) + g(x) = f(-x) + g(-x)$

$S(x)$ even $S(-x)$

so yes

odd

$S(-x) = f(-x) + g(-x) = -f(x) + -g(x)$
$= -f(x) - g(x)$
$= -(f(x) + g(x))$

odd function + odd function = odd function

even function + even function = even function

$S = f(x)g(x)$
$S(-x) = f(-x)g(-x)$
$= f(x)g(x)$
even

$S(-x) = f(-x)g(-x)$
$= (-f(x))(-g(x))$
$= f(x) \cdot g(x)$
$= S(x)$
$\Rightarrow$ even

①

$f(x) = 100 - 14x - 7x^2$

$= -7(x^2 + 2x) + 100$

$= -7(x^2 + 2x + 1) + 7 + 100$

$= -7(x+1)^2 + 107 \Rightarrow$ vertex: $(-1, 107)$

$\frac{-(-14)}{2(-7)} = -\frac{2}{2} = -1$

$x = \frac{-b}{2a}$

②

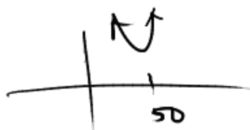
$m + n = 100 \Rightarrow m = 100 - n$

$m^2 + n^2$ is small as possible

so $(100-n)^2 + n^2 = 100 - 200n + 2n^2$

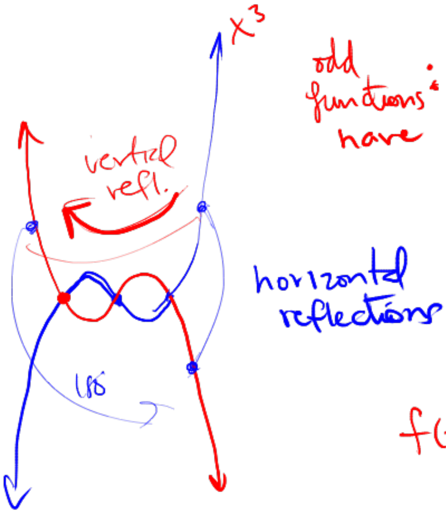
short cut: $\frac{-b}{2a} = \frac{-(-200)}{2(2)} = 50 = n$

parabola in the n-y plane:



$99^2 + 1$ vs $50^2 + 50^2$

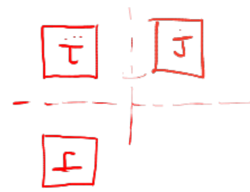
$100^2 = 10000$ and $2(50^2) = (2500)2 = 5000$



$f(-x) = -f(x)$

odd functions have: 180° rotational symmetry.

180° Rotation = Horizontal Reflection + Vertical Reflection



$f(x) \to f(-x)$

vertical reflection (same y's, symmetric to x-values)

$f(x) \to -f(x)$

horizontal flip/reflection - same x-values, y's reflected

odd's definition forces these two to be equal.

$f(x) = x^3 - x$

$f(2x) = (2x)^3 - 2x$

$= 8x^3 - 2x$

$2x(4x^2 - 1)$

$x = 0 \quad x = \pm 1/2$

$x^3 - x = 0$

$x(x^2 - 1) = 0$

$x = 0$

$x = \pm 1$

vs.

$f(\frac{1}{2}) = (\frac{1}{2}x)^3 - (\frac{1}{2}x)$

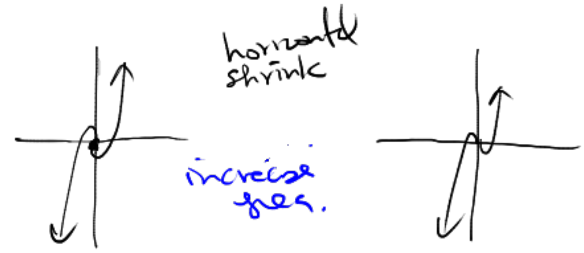
$= \frac{1}{8}x^3 - \frac{1}{2}x$

$= \frac{1}{2}x(\frac{1}{4}x^2 - 1)$

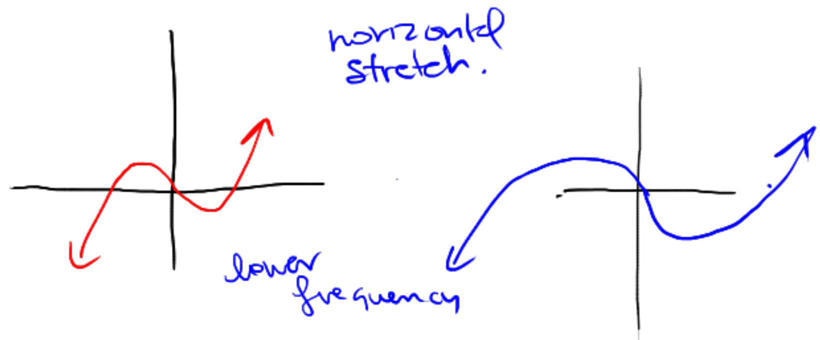
$x = 0$

$\frac{1}{4}x^3 = 1 \quad x^2 = 4 \Rightarrow x = \pm 2$

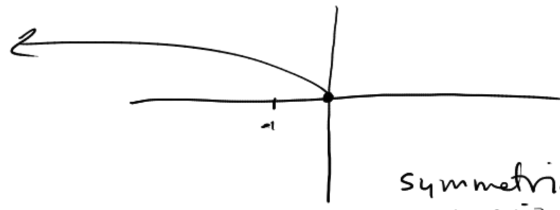
• so multiplying x by 2 did this



• Multiplying x by $\frac{1}{2}$ does this



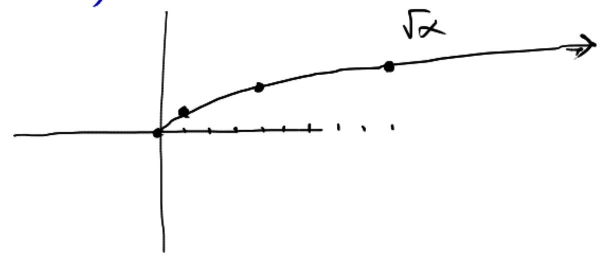
2. $g(x) = \sqrt{-x}$ domain of g is $(-\infty, 0]$



symmetric about y axis

$f(x) = \sqrt{x}$ domain of f is $[0, \infty)$

x	$\sqrt{x}$
0	0
1	1
4	2
9	3
16	4



$f(x) = x^{-2}$ even

$f(x) = \dfrac{1}{x^2}$

$f(-x) = \dfrac{1}{(-x)^2} = \dfrac{1}{x^2} = f(x)$

neg In → $f(-x)$; original → $\dfrac{1}{x^2}$

even

$f(x) = x^3 - x$

$f(-x) = (-x)^3 - (-x)$

$-x^3 + x = -(x^3 - x) = -f(x)$

neg In → $f(-x)$; original → $(x^3 - x)$; neg. orig → $-f(x)$

(odd)

$f(x) = -5x - 6$

(a) A.R.of C. b/w $(-1, f(-1))$, $(4, f(4))$ $\quad -5(4) - 6$

$(-1, -1)$, $(4, -26)$

$f(x_2)$ $f(x_1)$

$$\frac{y_2 - y_1}{x_2 - x_1}$$

(b) $(a, f(a))$, $(b, f(b))$

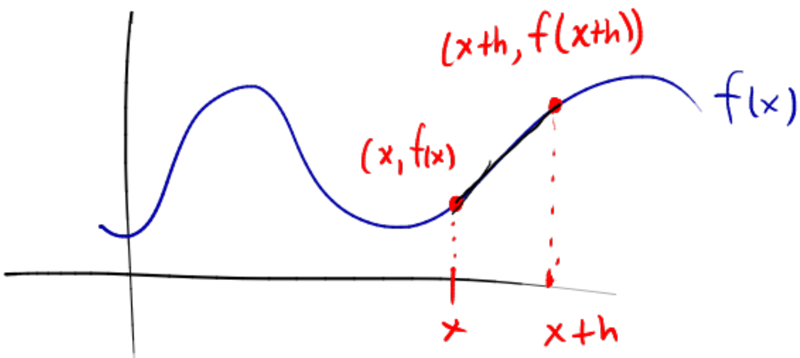
$$\frac{f(b) - f(a)}{b - a}$$

(c) $(x, f(x))$ & $(x+h, f(x+h))$

$x = x_1$, $f(x) = y_1$, $x+h = x_2$, $f(x+h) = y_2$

$$\frac{f(x+h) - f(x)}{h}$$

In Calculus, this "h" should be thought of as a small #.



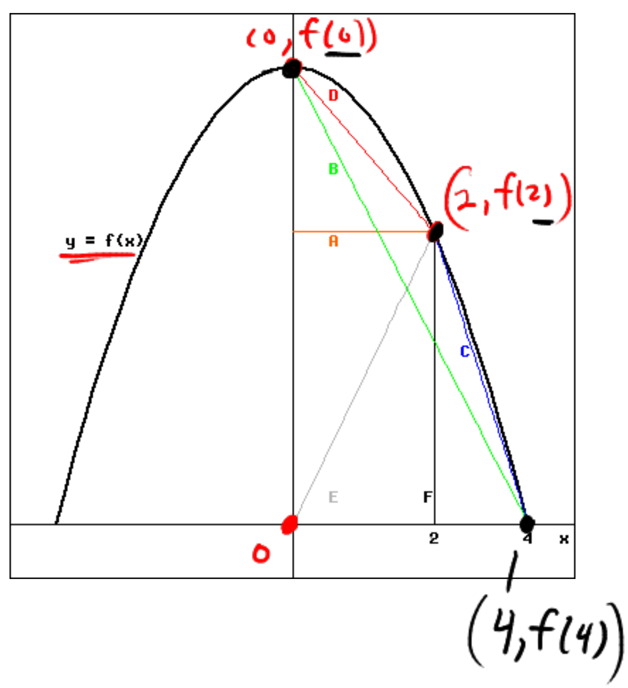
$$* \left(\frac{f(2) - f(0)}{2 - 0}\right) =$$

a slope, of the line connecting $(0, f(0))$ & $(2, f(2))$

Compute slope of line D.

$$\frac{f(0) - f(2)}{0 - 2} = \frac{f(2) - f(0)}{2 - 0}$$

$$\frac{f(4) - f(2)}{4 - 2}$$

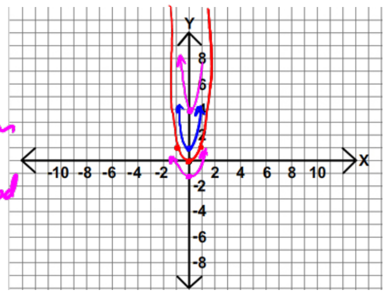


MA115 :: Section 2.4 :: Function Transformations and Max/Min

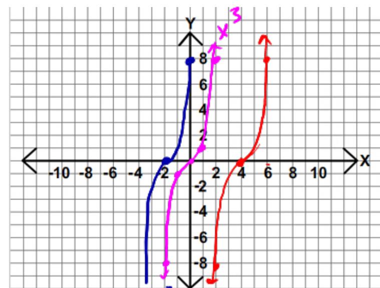
Sketch the transformations indicated in each caption. Scale the axes as appropriate.

x^6+1
x^6-1
x^6+4

see vertex move up/down shape unchanged



(a) 4 Vertical Shifts of $f(x) = x^6$

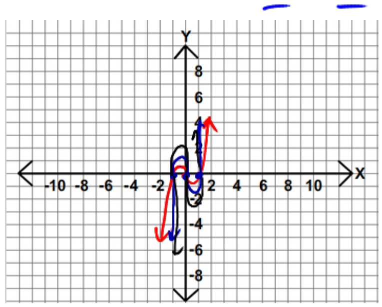


(b) 4 Horizontal Shifts of $f(x) = x^3$

see zeros move left/right shape unchanged

$(x+2)^3$

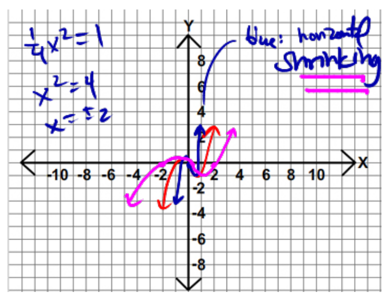
$(x-4)^3$



(c) 4 Vertical Stretch/Shrinkings of $f(x) = x^3 - x$

multiplication happens "outside"

$2(x^3-x)$



(d) 4 Horizontal Stretch/Shrinkings of $f(x) = x^3 - x$ → zeros are $x=0$, $x=\pm 1$

blue: horizontal shrinking

$\frac{1}{4}x^2=1$
$x^2=4$
$x=\pm 2$

$f(\frac{1}{2}x) = (\frac{1}{2}x)^3 - \frac{1}{2}x$
$= \frac{1}{8}x^3 - \frac{1}{2}x$
$= \frac{1}{2}x(\frac{1}{4}x^2-1)$

the zeros are $x=0$

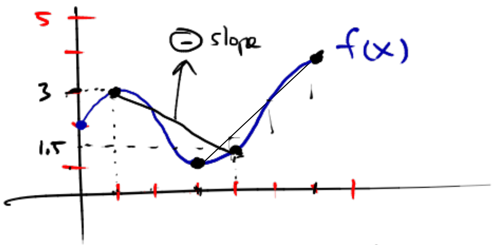
mult. is on inside.

subst. $x \mapsto 2x$

$f(2x) = (2x)^3 - (2x) = 8x^3 - 2x$
$= 2x(4x^2-1)$

my zeros of this function are $x=0$, $x=\pm 1/2$

How to compute A.R. of C. from graphs?



A.R. of C on [1, 4] (a, b)

$f(a)$ = height of graph above a. $= 3$

$f(b) = 1.5$

$\Rightarrow$ A.R. of C $= \frac{1.5 - 3}{4 - 1} = \frac{f(b) - f(a)}{b - a}$

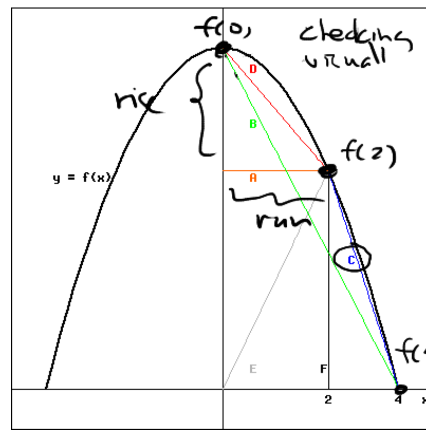
$= \frac{-1.5}{3} = \frac{-\frac{3}{2}}{\frac{3}{1}} = -\frac{3}{2} \cdot \frac{1}{3} = -\frac{1}{2}$

[3, 6] (a, b)

$f(3) = 1$

$f(6) = 4$

A.R.of: $\frac{4-1}{6-3} = \frac{3}{3} = 1$



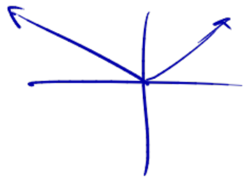
slope $= \frac{f(2) - f(0)}{2 - 0}$

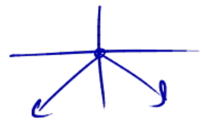
$\frac{1}{8}$ is the slope of line D

MA115 :: Section 2.4 :: Function Transformations and Max/Min

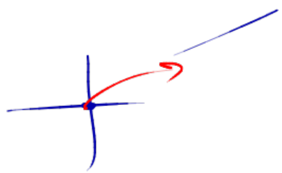
1. Sketch $f(x) = |x|$ and $g(x) = -|x|$

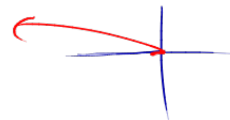
outside mult. by neg ⟷ flipping / reflect across x-axis





2. Sketch $f(x) = \sqrt{x}$ and $g(x) = \sqrt{-x}$.



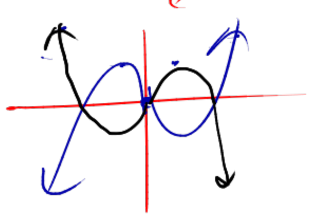


inside mult. by neg. (−) reflection across y-axis

3. A function is even if $f(-x) = f(x)$ and is odd if $f(-x) = -f(x)$. Indicate whether the following functions are even, odd or neither.

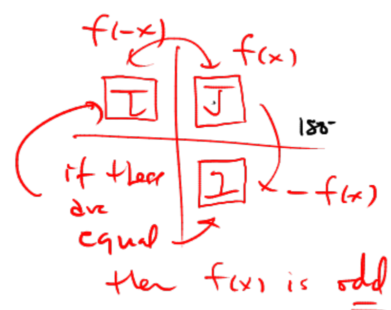
$\frac{1}{(-x)^2} = \frac{1}{x^2}$ even

$f(x) = x^{-2}$	even	$f(x) = x^2 + x$	neither
$f(x) = x^3 - x$	odd	$f(x) = x + \frac{1}{x}$	odd
$f(x) = x^4 - 3x^2$	even	$f(x) = -x$	odd.



$$(-x)^3 - (-x) = -x^3 + x = -(x^3 - x)$$

original



4. Is the sum (or prduct) of **any** two even functions even, odd or neither?

Pretend $f(x)$ & $g(x)$ are even.

So $f(-x) = f(x)$, $g(-x) = g(x)$

define $s(x) = f(x) + g(x)$ then $s(-x) = f(-x) + g(-x)$

$= f(x) + g(x)$

$= s(x)$

$f(x) = -f(-x)$ true if f is odd. ($-f(x)$)

same for odd: Here I'm pretending f, g odd.

$s(x) = f(x) + g(x)$

$s(-x) = f(-x) + g(-x)$

$= -f(x) - g(x) = -(f(x) + g(x)) = -s(x) \Rightarrow s(x)$ is odd

5. Is the sum (or product) of **any** two odd functions odd, odd or neither?

product even

f is odd .... $f(-x) = -f(x)$

g is odd .... $g(-x) = -g(x)$

define

$m(x) = f(x) \cdot g(x)$

check

$m(-x) = f(-x) \cdot g(-x)$

$-f(x) \cdot (-g(x))$

$= f(x) \cdot g(x) = m(x)$

so $m(x)$ is even

Confused? look at this example!

$f(x) = x^3$, $g(x) = x$ (both odd)

$f(x) \cdot g(x) = x^4$

6. A general quadratic $f(x) = ax^2 + bx + c$ can be expressed in **standard form**

$$f(x) = a(x-h)^2 + k$$

by completing the square. The graph of f is a parabola with **vertex** (h, k), and the parabola opens **upward if** $a > 0$ and **downward if** $a < 0$. If $a > 0$, then f will have a minimum value $k = f(h)$. If $a < 0$, then f will have a maximum value $k = f(h)$. The algorithm to complete the square is as follows:

$$\left(\frac{1}{2}\frac{b}{a}\right)^2 = \frac{b^2}{4a^2}$$

1. $f(x) = ax^2 + bx + c$

2. $= a\left(x^2 + \frac{b}{a}x\right) + c$

3. $= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - a\left(\frac{b^2}{4a^2}\right)$ (compensation)

$= a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$ (stand. form)

occurs here (h), k → max. value

Complete the square to **find the maximum value of**

$$f(x) = 100 - 14x - 7x^2.$$

$$-7(x^2+2x) + 100$$

$$-7(x^2+2x+1) + 100 + 7$$

$$-7(x+1)^2 + 107$$

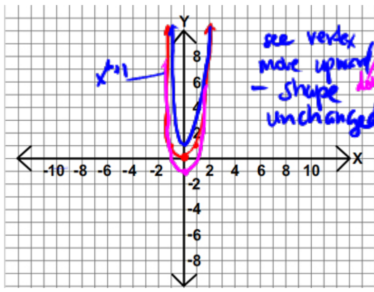
max. value = 107

occurs @ $x = -1$

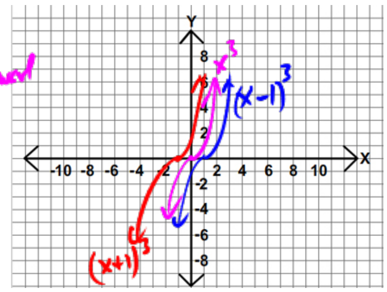
MA115 :: Section 2.4 :: Function Transformations and Max/Min

Sketch the transformations indicated in each caption. Scale the axes as appropriate.

vertical shifts
x^6+1
x^6-1



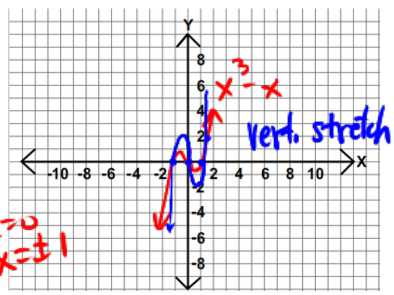
(a) 4 Vertical Shifts of $f(x) = x^6$



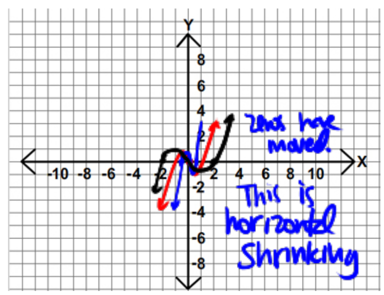
(b) 4 Horizontal Shifts of $f(x) = x^3$

$(x-1)^3$ Rt.

$(x+1)^3$ left



(c) 4 Vertical Stretch/Shrinkings of $f(x) = x^3 - x$



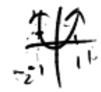
(d) 4 Horizontal Stretch/Shrinkings of $f(x) = x^3 - x$

Horizontal stretch
$f(x) = x^3 - x$
$f(\frac{1}{2}x) = \frac{1}{8}x^3 - \frac{1}{2}x$
$\frac{1}{2}x(\frac{1}{4}x^2 - 1)$ zeros: $x = 0$
$\frac{1}{4}x^2 = 1 \Rightarrow x^2 = 4$
$x = \pm 2$

zeros: $x = 0$, $x = \pm 1$

$2f(x) = 2(x^3 - x)$ vs $(2x)^3 - (2x) = f(2x) = 8x^3 - 2x = 2x(4x^2 - 1)$ zeros $x = 0$, $x = \pm\frac{1}{2}$

mult. is on outside — mult on inside



→ means $f(-x) = f(x)$

4. Is the sum (or prduct) of **any** two even functions even, odd or neither?

Pretend $f(x)$ is even, $g(x) =$ is even

$S(x) = f(x) \cdot g(x)$ create new function

test whether $s(x)$ is even / odd.

$s(-x) = f(-x)\, g(-x) =$ ⇒ S is **even**

$= f(x) \cdot g(x) = S(x)$

ex. x^2 even

$+ x^4 + x^6$ even

$x^6 + x^4 + x^2$ even

5. Is the sum (or product) of **any** two odd functions odd, odd or neither?

Pretend $f(x)$ & $g(x)$ are odd. Determine whether

$S(x) = f(x) \cdot g(x)$ is even / odd / neither?

$s(-x) = f(-x)\, g(-x)$

$= [-f(x)] \cdot [-g(x)]$

$= f(x)\, g(x) = s(x)$ (orig) ⇒ even.

ex. $x^3 \cdot x^5 = x^8$

$x^3 + x^5$ is odd

$A(x) = f(x) + g(x)$ when they're odd

A is odd

$A(-x) = f(-x) + g(-x) =$

END

start $= -f(x) - g(x) = -[f(x) + g(x)] = -A(x)$

6. A general quadratic $f(x) = ax^2 + bx + c$ can be expressed in **standard form**

$$f(x) = a(x-h)^2 + k$$

by completing the square. The graph of f is a parabola with **vertex** (h, k), and the parabola opens **upward if** $a > 0$ and **downward if** $a < 0$. If $a > 0$, then f will have a minimum value $k = f(h)$. If $a < 0$, then f will have a maximum value $k = f(h)$. The algorithm to complete the square is as follows:

- $$\begin{aligned} f(x) &= ax^2 + bx + c \\ &= a\left(x^2 + \frac{b}{a}x\right) + c \\ &= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - a\left(\frac{b^2}{4a^2}\right) \\ &= a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a} \end{aligned}$$

chosen to make a perfect square: $\left(\frac{1}{2}\frac{b}{a}\right)^2 = \frac{b^2}{4a^2}$

Complete the square to **find the maximum value of**

$$f(x) = 100 - 14x - 7x^2.$$

$\left(\frac{7}{2}\right)^2 = \frac{49}{4}$

$-7x^2 - 49x + 100$

$-7\left(x^2 + 7x + \frac{49}{4}\right) + 100 - \frac{49}{4}(-7)$

$-7\left(x + \frac{7}{2}\right)^2 + 185.75$

x-value of vertex; y-value of vertex

vertex: $\left(-\frac{7}{2}, 185.75\right)$ → max value

$= (-7x^2 - 14x) + 100$

$= -7(x^2 + 2x) + 100$

$= -7(x^2 + 2x + 1) + 100 - 1(-7)$

$= -7(x+1)^2 + 107$

$\Rightarrow$ vertex is $(-1, 107)$ → max value

max value occurs at $x = -1$

6. A general quadratic $f(x) = ax^2 + bx + c$ can be expressed in **standard form**

$$f(x) = a(x-h)^2 + k$$

by completing the square. The graph of f is a parabola with **vertex** (h, k), and the parabola opens **upward if** $a > 0$ and **downward if** $a < 0$. If $a > 0$, then f will have a minimum value $k = f(h)$. If $a < 0$, then f will have a maximum value $k = f(h)$. The algorithm to complete the square is as follows:

$$\begin{aligned} f(x) &= ax^2 + bx + c \\ &= a\left(x^2 + \frac{b}{a}x\right) + c \\ &= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - a\left(\frac{b^2}{4a^2}\right) \\ &= a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a} \end{aligned}$$

Complete the square to **find the maximum value of**

$$f(x) = 100 - 49x - 7x^2.$$

Plan: ① put into standard form, ② read off the vertex ③ max value = y-value of vertex

$f(x) = (-7x^2 - 49x) + 100$

$= -7(x^2 + 7x \quad) + 100$

↑ insert here $\left(\frac{7}{2}\right)^2$ ↑ compensate for doing so $-\left(\frac{7}{2}\right)^2(-7)$

$\frac{49}{4} \cdot 7 = \frac{7^3}{4}$

$= -7\left(x^2 + 7x + \left(\frac{7}{2}\right)^2\right) + 100 + \frac{7^3}{4}$

$= -7\left(x - -\frac{7}{2}\right)^2 + 185.75$

vertex: always (h, k)

$a(x-h)^2 + k$

constant

vertex: $\left(-\frac{7}{2}, 185.75\right)$ → max. value of $f(x)$.

7. The algorithm above implies that the maximum value of $f(x) = ax^2 + bx + c$ is $f\left(-\frac{b}{2a}\right)$. Use this to find two integers whose sum is 100 and whose sum of squares is a minimum.

$50^2 = 2500$

m, n two int's

$m + n = 100 \Rightarrow m = 100 - n$

want: $m^2 + n^2$ to be as small as possible.

so $(100-n)^2 + n^2$ is equivalent to

$= 100^2 - 200n + 2n^2$

$= (2n^2 - 200n) + 100^2$

insert: compensate

$= 2(n^2 - 100n + 2500) + 100^2 - 2500(2) = 2(n-50)^2 + 5000$

vertex: $(50, 5000)$ → min value

→ $n = 50$, so $m = 50$

$100^2 + 0^2 = 100^2 = 10{,}000$

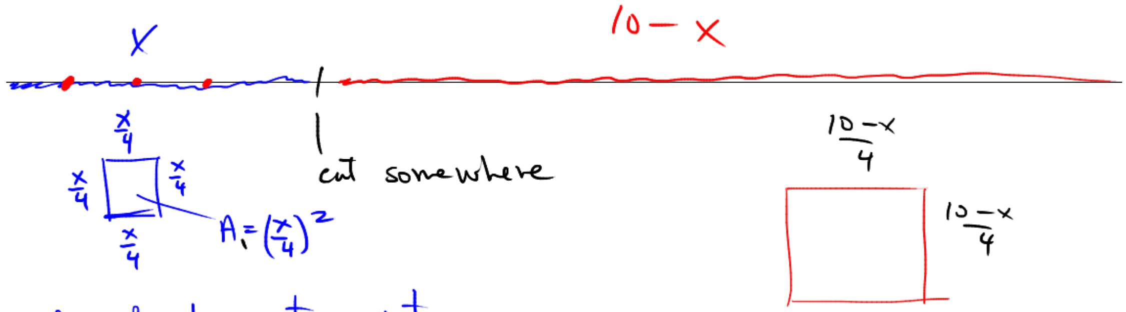
or

$99^2 + 1^2 = 9802$

8. Find the maximum value of the function

$$f(x) = 3 + 4x^2 - x^4.$$

You have a wire, 10' long



$A_2 = \left[\frac{10-x}{4}\right]^2$

goal: find cut point so that the combined areas is as small as possible

$f(x) = ax^2+bx+c$

vertex: $\left(-\frac{b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$A_1 + A_2 = \left(\frac{x}{4}\right)^2 + \left(\frac{10-x}{4}\right)^2 = \frac{x^2}{16} + \frac{100 - 20x + x^2}{16}$

$= \frac{2x^2 - 20x + 100}{16} = \frac{1}{16}\left(2x^2 - 20x + 100\right)$

$= \frac{2}{16}\left(x^2 - 10x + 50\right)$

$= \frac{2}{16}(x^2-10x) + \frac{2}{16}(50)$

$= \frac{2}{16}\left(x^2 - 10x\right) + \frac{25}{4}$

now insert 25 — compensate

$= \frac{1}{8}\left(x^2 - 10x + 25\right) + \frac{25}{4} - 25\left(\frac{1}{8}\right)$

$= \frac{1}{8}(x-5)^2 + \frac{25}{8}$

$a = \frac{2}{16} = \frac{1}{8}$

$b = \frac{-20}{16}$

what is $\frac{-b}{2a}$

$= \frac{-\left(\frac{-20}{16}\right)}{2\left(\frac{1}{8}\right)} = 5$

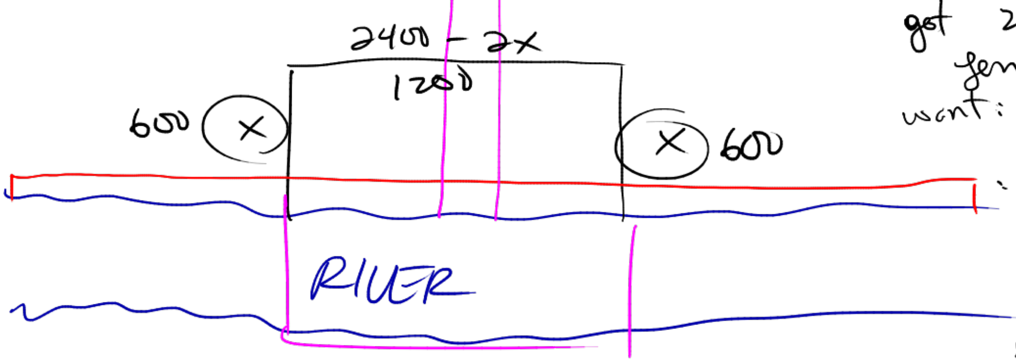
$\frac{2 \cdot 50}{16} = \frac{100}{16} = \frac{25}{4}$

$25\left(\frac{1}{4} - \frac{1}{8}\right) = 25\left(\frac{1}{8}\right) = \frac{25}{8} \approx 3.2$

$x = 5$!!!

got 2400' of fence.

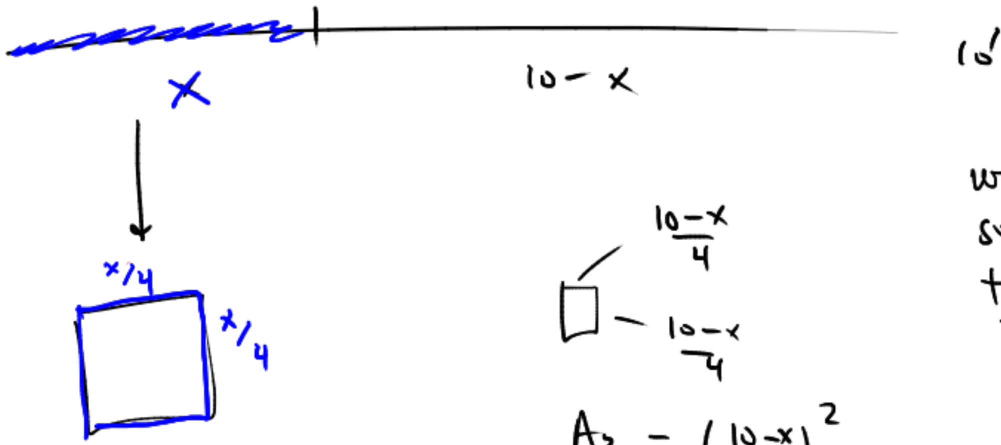
want: rectangular fence
: borders river (3 side)
: biggest area as possible.



$A = x(2400 - 2x)$

$= 2400x - 2x^2$ .....

$x = \frac{-2400}{2(-2)} = 600$



where to cut so sum of the areas is as small as possible.

$A_1 = \left(\frac{x}{4}\right)^2$

$A_2 = \left(\frac{10-x}{4}\right)^2$

$$A_1 + A_2 = \frac{x^2}{16} + \frac{100 - 20x + x^2}{16} = \frac{1}{16}\left(2x^2 - 20x + 100\right)$$

$$= \frac{1}{8}\left(x^2 - 10x + 50\right) \quad (*)$$

Recall: if $f(x) = ax^2 + bx + c$ the extreme value / vertex is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

$a = \frac{1}{8}$

$b = -\frac{10}{8} = \frac{-5}{4}$

so $\frac{-(-5/4)}{2(1/8)} = \frac{5/4}{1/4} = 5$

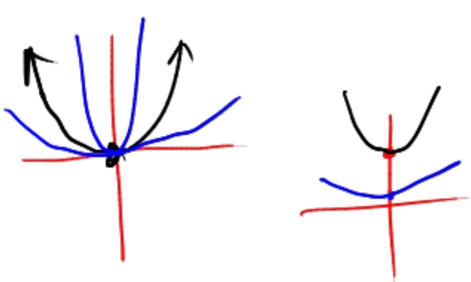
$$= \frac{1}{8}\left(x^2 - 10x\right) + \frac{50}{8}$$

$$= \frac{1}{8}\left(x^2 - 10x + 25\right) + \frac{50}{8} - 25\left(\frac{1}{8}\right)$$

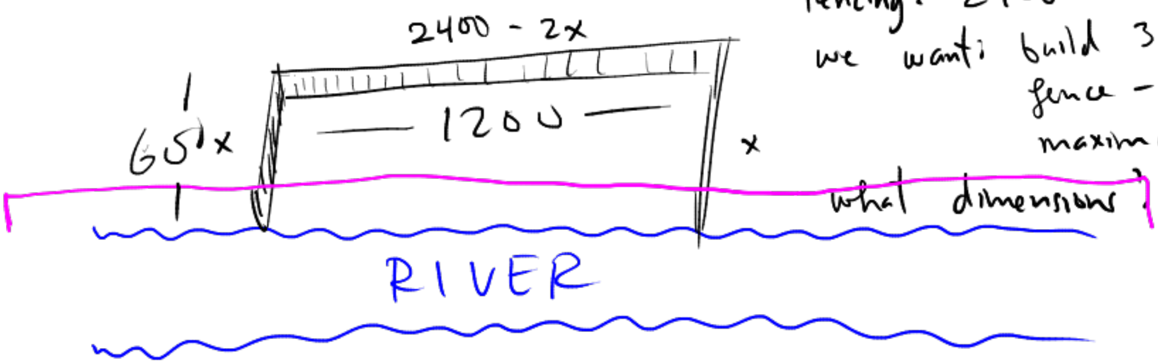
$$= \frac{1}{8}(x-5)^2 - \frac{25}{8}$$

vertex: $\left(5, \frac{25}{8}\right)$

5 is where to cut.



Fencing: 2400'
we want: build 3 sided fence - with maximum area
what dimensions?



Area = length * width = $(2400-2x)x = 2400x - 2x^2$

$a = -2$, $b = 2400$ the vertex is

$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$= \left(\frac{-2400}{2(-2)}\right.$

$= (600, \quad)$

$\uparrow$ $2400(600) - 2(600)^2$

So
w = 600

9. (This one's fun.) Complete the square of $f(x) = ax^2 + bx + c$, then set equal to zero and solve for x to **derive** the quadratic formula.

given $(ax^2 + bx) + c = 0$ — a, b, c constants & $a \neq 0$.

$a\left(x^2 + \frac{b}{a}x\right) + c = 0$ (compensate; add here)

add here: $\left(\frac{\left(\frac{b}{a}\right)}{2}\right)^2 = \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$

$$a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c \left(-\frac{b^2}{4a^2}\right)a = 0$$

$$a\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2}{4a} + c\right) = 0$$

$$\frac{1}{a}\left(a\left(x + \frac{b}{2a}\right)^2\right) = \left(\frac{b^2}{4a} - c\right)\frac{1}{a}$$

This is why the quadratic formula works.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ca}{4aa} = \frac{b^2 - 4ac}{4a^2}$$

square root

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \boxed{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$\frac{90-31}{90}$ = grade.

−31 *

Name: **MA115 :: Exam 1**

For full credit, circle your answers and show all your work!

Part A

1. Solve the inequality $-4|2-3x|-6<13$

$-4|2-3x| < 19 \Rightarrow |2-3x| > -\frac{19}{4}$

$\frac{19}{4} > 2-3x > -\frac{19}{4}$

$\left(-\frac{1}{3}\right)\frac{11}{4} > -3x > -\frac{27}{4}\left(-\frac{1}{3}\right)$

$-\frac{11}{12} < x < \frac{9}{4}$

2. Solve the inequality

$$\frac{x^3-4x^2}{x+5} \geq 0$$

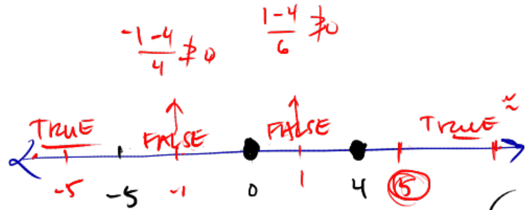
Zeros: $x^3-4x^2=0$

$x^2(x-4)=0$

$x=0,\ x=4$

Breaks: $x+5=0$

$x=-5$



$\frac{x^3}{x} = x^2$

"the single point $x=0$"

$(-\infty,-5)\cup\{0\}\cup[4,\infty)$

3. Simplify the expression and eliminate any negative exponents:

$$\left(\frac{2x^{3/2}y^3}{x^2y^{-1/2}}\right)^{-2} = \left(\frac{x^2y^{-1/2}}{2x^{3/2}y^3}\right)^2$$

$$= \frac{x^4y^{-2/2}}{4x^{6/2}y^6} = \frac{x^4y^{-1}}{4x^3y^6} = \frac{x}{4y^7}$$

Name: **MA115 :: Exam 1**

For full credit, circle your answers and show all your work!

Part A

1. Solve the inequality $-4|2-3x|-6<13$

$-4|2-3x|<19 \Rightarrow |2-3x|>\frac{19}{-4}$

$\frac{19}{4}>2-3x>\frac{-19}{4}$

$\left(\frac{-1}{3}\right)\frac{11}{4}>-3x>\frac{-27}{4}\left(\frac{-1}{3}\right)$

$\boxed{\frac{-11}{12}<x<\frac{9}{4}}$

2. Solve the inequality $\frac{x^3-4x^2}{x+5}\geq 0$

non-linear

Zeros: $x^3-4x^2=0$

$x^2(x-4)=0$

$x=0,\ x=4$

Breaks: $x+5=0$

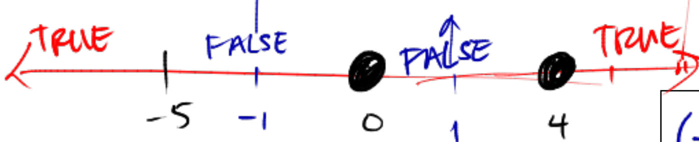
$x=-5$

$\frac{-1-4}{4}\not\geq 0$ $\qquad$ $\frac{1-4}{1+5}\not\geq 0$

acts like this

$\approx \frac{x^3}{x}=x^2\geq 0$

for x w/ large abs. value



$\boxed{(-\infty,-5)\cup\{0\}\cup[4,\infty)}$

3. Simplify the expression and eliminate any negative exponents:

$$\left(\frac{2x^{3/2}y^3}{x^2y^{-1/2}}\right)^{-2}$$

Two things! Flip & square.

$\left(\frac{x^2y^{-1/2}}{2x^{3/2}y^3}\right)^2$ multiply exponents

$\frac{x^4y^{-2/2}}{2^2x^{6/2}y^6}=\frac{x^4y^{-1}}{4x^3y^6}=\boxed{\frac{x}{4\cdot y^7}}$

common error: $\frac{y^2 - x^2}{y - x} = y - x$

Name: **MA115 :: Exam 1**

4. Simplify the expression below:

$$\frac{\frac{y}{x} - \frac{x}{y}}{\frac{1}{x} - \frac{1}{y}}$$

$\frac{y^2 - x^2}{yx} \cdot \frac{yx}{y - x} = \frac{y^2 - x^2}{y - x} = \frac{(y-x)(y+x)}{(y-x)} = y + x$

$\frac{\frac{y^2 - x^2}{yx}}{\frac{y-x}{yx}}$ flip & multiply

5. Rationalize the denominator:

$$\frac{y}{\sqrt{3} + \sqrt{y}}$$

signs must be opposite

$\left(\frac{\sqrt{3} - \sqrt{y}}{\sqrt{3} - \sqrt{y}}\right) = \boxed{\frac{y(\sqrt{3} - \sqrt{y})}{3 - y}}$

$(\sqrt{3})(-\sqrt{y}) + \sqrt{y}\sqrt{3} = 0$

$3/2 - (-1/2) = 4/2 = 2 \quad \| \quad \frac{1}{2} - (-\frac{1}{2}) = 1$

6. Factor the expression completely and simplify your answer. Write your answer with positive exponents.

$$3x^{3/2} - 9x^{1/2} + 6x^{-1/2}$$

look for common term with smallest exponent

Equal

$x^{-1/2}\left(3x^2 - 9x + 6\right)$

$3x^{-1/2}(x^2 - 3x + 2)$

$(x-2)(x-1)$

2

$\boxed{\frac{3(x-2)(x-1)}{x^{1/2}}}$

$$\frac{5+2}{2+12} = \frac{1}{2} \qquad \frac{x^2+x}{x-4}$$

7. Perform the indicated operations and simplify:

$$\frac{x^2}{x^2-4} - \frac{x+1}{x+2}\cdot\frac{(x-2)}{(x-2)}$$

(x^2-x-2)

$(x+2)(x-2)$

$$\frac{x^2 - x^2 + x + 2}{(x^2-4)} = \frac{x+2}{x^2-4} = \frac{1}{x-2}$$

8. Factor the expression completely.

$$[x^3 + 3x^2] + [4x + 12]$$

$$x^2(x+3) + 4(x+3)$$

new common factor

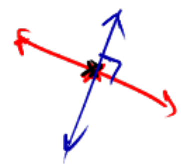
$$(x+3)(x^2+4)$$

9. Find all solutions to the equations:

$$2x^4 + 18x^2 - 54 = 0$$

Part B

10. The line segment between $(-1, 1)$ and $(4, -2)$ is called AB. Find the equation of the line that intersects AB at its midpoint and is perpendicular to AB.



Find midpoint: add points, divide by 2. $\left(\frac{3}{2}, \frac{-1}{2}\right)$

slope of red line: $\frac{\text{rise}}{\text{run}} = \frac{-2-1}{4-(-1)} = \frac{-3}{5}$

$-\frac{15}{6} - \frac{3}{6} = \frac{-18}{6} = -3$

Blue slope $= \frac{5}{3}$

$y - \left(\frac{-1}{2}\right) = \frac{5}{3}\left(x - \frac{3}{2}\right)$

$y = \frac{5}{3}x - 3$

11. What quantity of a 60% acid solution must be mixed with a 50% acid solution to produce 300 mL of a 50% acid solution?

$.6(x) + .5(300 - x) = 300(.50)$

$.6x + 150 - .5x = 150$

$.1x = 0 \Rightarrow x = 0$

$\frac{90-31}{90}$ = grade.

-31

4. Simplify the expression below:

$$\frac{\frac{y}{x}-\frac{x}{y}}{\frac{1}{x}-\frac{1}{y}}$$

$= \dfrac{\frac{y^2-x^2}{yx}}{\frac{y-x}{yx}}$

$\frac{y^2-x^2}{yx}\cdot\frac{yx}{y-x} = \frac{y^2-x^2}{y-x}$

$\frac{(y-x)(y+x)}{(y-x)}$

$y+x$

5. Rationalize the denominator:

$$\frac{y}{\sqrt{3}+\sqrt{y}}$$

$\cdot \frac{\sqrt{3}-\sqrt{y}}{\sqrt{3}-\sqrt{y}} = \frac{y(\sqrt{3}-\sqrt{y})}{3-y}$

$\sqrt{3}(-\sqrt{y}) + \sqrt{y}\sqrt{3}$ "0"

6. Factor the expression completely and simplify your answer. Write your answer with positive exponents.

common term w/ smallest exponent $x^{-1/2}$

$$3x^{3/2} - 9x^{1/2} + 6x^{-1/2}$$

$\frac{3}{2} - (-\frac{1}{2}) = \frac{4}{2} = 2$

$\frac{1}{2} - (-\frac{1}{2}) = 1$

Equal

$x^{-1/2}[3x^{2} - 9x^{1} + 6]$

$3x^{-1/2}(x^2 - 3x + 2)$

$3x^{-1/2}(x-2)(x-1)$

$\frac{3(x-2)(x-1)}{x^{1/2}}$

$\frac{90-31}{90}$ = grade.

-31*

Name: **MA115 :: Exam 1**

7. Perform the indicated operations and simplify:

$$\frac{x^2}{x^2-4} - \frac{x+1}{x+2}$$

$(x-2)(x+2)$

$x^2 - x - 2$

$$\frac{x^2 - x^2 + x + 2}{x^2 - 4} = \frac{x+2}{x^2-4} = \frac{x+2}{(x-2)(x+2)} = \boxed{\frac{1}{x-2}}$$

8. Factor the expression completely.

$$x^3 + 3x^2 + 4x + 12$$

$x^2(x+3) + 4[x+3]$

$(x+3)(x^2+4)$

9. Find all solutions to the equations:

$$2x^4 + 18x^2 + 54 = 0$$

$(w = x^2)$

quadratic.

$2w^2 + 18w + 54 = 0$

$2(w^2 + 9w + 27) = 0$

$w = \frac{-9 \pm \sqrt{81 - 4 \cdot 27}}{2}$

$= \frac{-9 \pm \sqrt{-27}}{2}$

w is not real $\Rightarrow x^2, x$ not real

no real solutions

$\frac{90-31}{90}$ = grade. −31

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Part B

10. The line segment between $(-1, 1)$ and $(4, -2)$ is called AB. Find the equation of the line that intersects AB at its midpoint and is perpendicular to AB.

Midpoint: $\left(\frac{-1+4}{2}, \frac{1-2}{2}\right) = \left(\frac{3}{2}, \frac{-1}{2}\right)$

$\frac{-15}{6} - \frac{3}{6} = \frac{-18}{6}$

$m = \frac{-2-1}{4-(-1)} = \frac{-3}{5}$

$m_\perp = \frac{5}{3}$

$y - \left(\frac{-1}{2}\right) = \frac{5}{3}\left(x - \frac{3}{2}\right)$

$$y = \frac{5}{3}x - 3$$

11. What quantity of a 60% acid solution must be mixed with a 50% acid solution to produce 300 mL of a 50% acid solution?

$.60x + .50(300 - x) = .50(300)$

$.6x + 150 - .5x = 150$

$-150 \qquad -150$

$.1x = 0$

$x = 0$