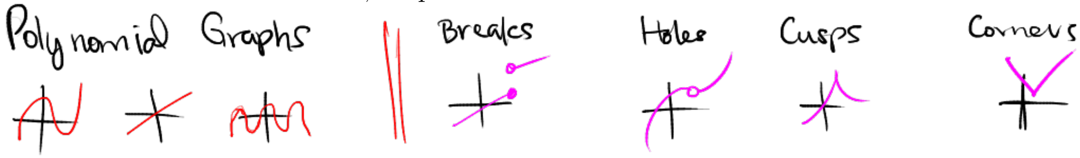
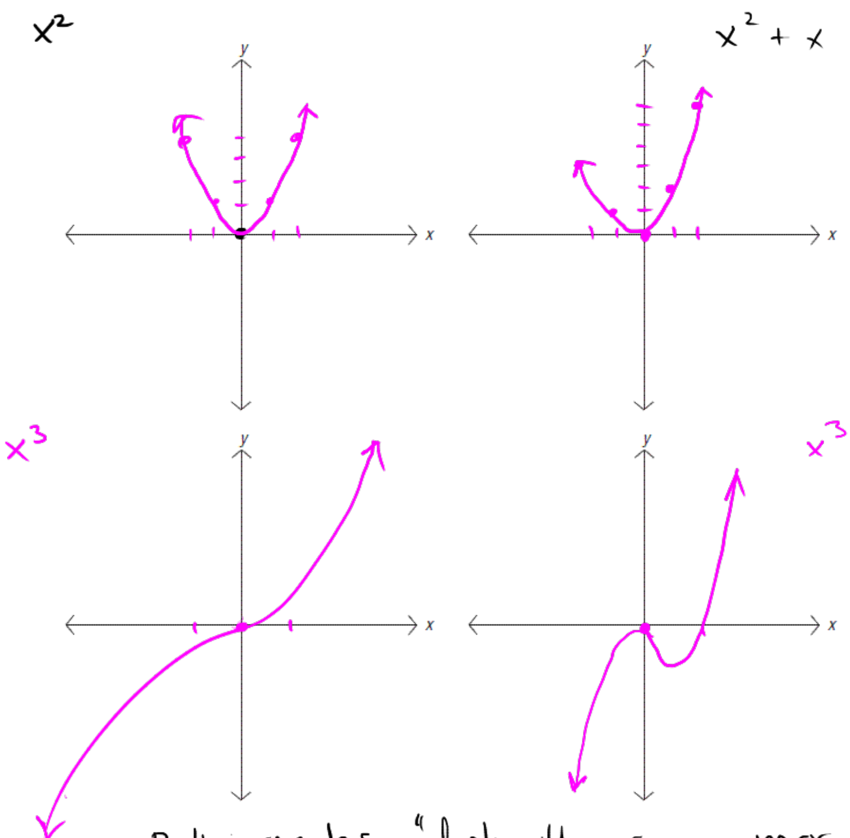
1. Graphs of polynomials are smooth and continuous, i.e., no breaks, holes, cusps or corners.



2. A polynomial has a *degree, coefficients, a constant term, leading coefficient, leading term*

Ex. $f(x) = 3x^4 + 5x^2 + 1$ ... f has degree of 4, coefficients 3, 5, 1, constant term = 1, leading coefficient = 3

3. The leading term governs the *end behavior.*



$x^3 - x^2$

$= x^2(x-1)$

Both Graphs are heading in same directions at their respective ends

Both graphs "look the same near the ends"

End Behavior: Plug in BIG positive numbers. Are the outputs growing to infinity, -infinity, or settling down to some stationary number? Repeat for BIG negatives.

As a result of the nature of exponential growth, only the first term (and its coefficient) are relevant to the question of End Behavior.

Example: Describe the end behavior of $x^4 + 3000x^3 + x^2 + 1$

As $x \to \infty$, $f(x) \to \infty$ $(1000^4$ is BIG!)

As $x \to -\infty$, $f(x) \to \infty$ $(-1000)^4$ is BIG!

Example: Describe the end behavior of $x^5 + 3000x^4 + x^2 + 1$

As $x \to \infty$, $f(x) \to \infty$ $(1000)^5$ is BIG

As $x \to -\infty$ $f(x) \to -\infty$ $(-1000)^5$ is a "BIG NEGATIVE"

Real Zeros of Polynomials

The following are equivalent.

Example: $P(x) = x^3 - 2x^2 - x + 2$

1. c is a zero of P.

$P(1) = (1) - 2(1) - 1 + 2 = 0$

$P(-1) = -1 - 2 + 1 + 2 = 0$

$P(2) = 8 - 8 - 2 + 2 = 0$

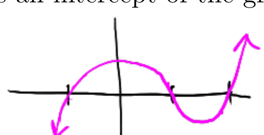
2. $x = c$ is a solution of the equation $P(x) = 0$.

from above $\Rightarrow$ $P(1) = 0$
$P(-1) = 0$
$P(2) = 0$

3. $x - c$ is a factor of $P(x)$.

$P(x) = (x-1)(x+1)(x-2)$

4. $x = c$ is an intercept of the graph of P.

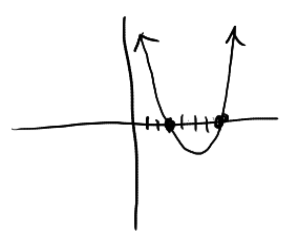


Example: $P(x) = x^2 - 10x + 21$.

$P(x) = (x-3)(x-7)$

$P(3) = 0$

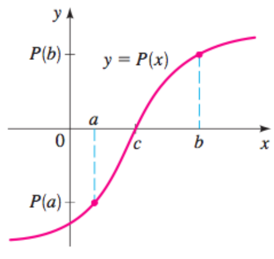
$P(7) = 0$



Intermediate Value Theorem for Polynomials

If P is a polynomial function and $P(a)$ and $P(b)$ have opposite signs, then there is at least one value c betwenn a and b for which $P(c) = 0$.

Example: If your height were given as a polynomial, and in 2001 you were 4 feet tall, and in 2010 you were 6 feet tall. This theorem guarantees that some precise time between 2001 and 2010, you were 5 feet tall.



So, in the example above, we found the zeros of the polynomial. The IVT tells us that all the $x - values$ BETWEEN the zeros give $y - values$ with the same sign. This helps to graph the function.

GUIDELINES FOR GRAPHING POLYNOMIAL FUNCTIONS

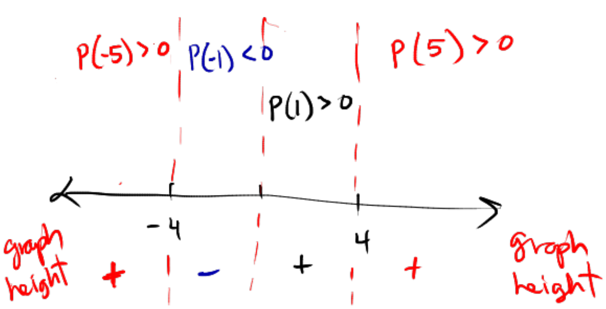
1. **Zeros** Factor, and find all real zeros. These are the x-intercepts.
2. **Test Points** Choose points in the regions created by the zeros to test whether the poly is positive or negative in that region.
3. **End Behavior** Determine the end behavior
4. Plot the intercepts and the test points. Sketch smooth curve passing through all points and which exhibits the required end behavior.

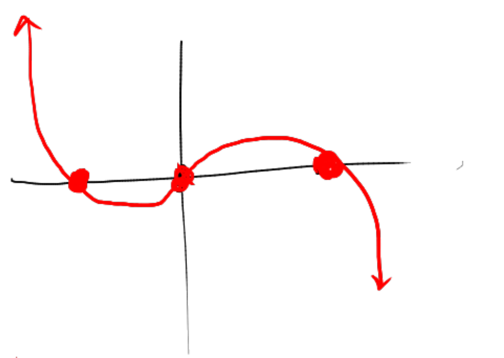
Graph the following by hand. Example: $P(x) =$ [illegible] $= 16x - x^3$

$P(x) = -x(x-4)(x+4)$

zeros:

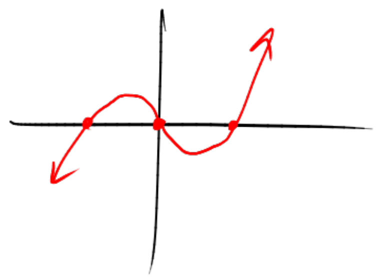
$x=0, x=\pm 4$





Example: $P(x) = x^3 - x$. $= x(x^2-1) = x(x-1)(x+1)$

very similar to previous one ... just opposite end behavior & small change in zeros



Example: $P(x) = x^4 - 109x^2 + 900$.

degree 4 + leading term positive = happy parabola

$= (x^2-100)(x^2-9)$

$= (x-10)(x+10)(x-3)(x+3)$