$\frac{x+3}{3x+10}$

MA115 :: Section 2.8 :: Inverse Functions

Warm-up 1: If $f(g(x)) = 6(x^7 + 7)^9$ and $g(x) = x^7 + 7$, find $f(x)$.

$f(x) = 6x^9$

$g\left(\frac{1}{x+3}\right) = \frac{1}{\boxed{\left(\frac{1}{x+3}\right) + 3}}$ " use (A)

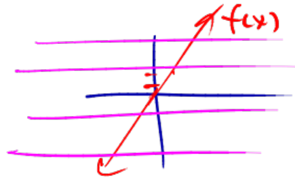
Warm-up 2: If $f(x) = x + 3$ and $g(x) = \frac{1}{x+3}$, find

$(f \circ g)(2)$__________ and $(g \circ g)(x)$__________

$f(g(x)) = f\left(\frac{1}{x+3}\right) = \boxed{\frac{1}{x+3} + 3}\left(\frac{x+3}{x+3}\right) = \frac{1 + 3x + 9}{x+3} = \frac{3x + 10}{x+3}$ (A)

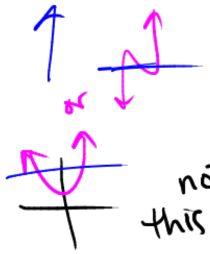
$f(g(2)) = \frac{6+10}{2+3} = \frac{16}{5} = 3.2$

(A) #4 satisfies the horiz line test.



Ex. $y = 2x$ as a function.
$f(x) = 2x$
what's the inverse function? f^{-1} not $\frac{1}{f}$

don't satisfy V.L.T



not this ←

not (1-1)

MA115 :: Section 2.8 :: Inverse Functions

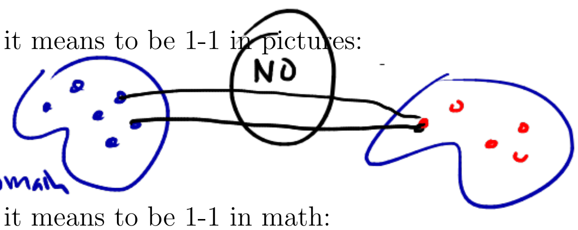
$f^{-1}(f(x)) = x$

For **certain** functions, there is an equal and opposite function, an *inverse function*. This function "undoes" whatever the function has done. Functions that have inverses are called *one-to-one*.

Here: $g(x) = \frac{1}{2}x$

$g(f(x))$
$= \frac{1}{2}(2x) = x$

1. What it means to be 1-1 in words: *never take on the same value twice*
2. What it means to be 1-1 in pictures:



Domain

Range

3. What it means to be 1-1 in math:

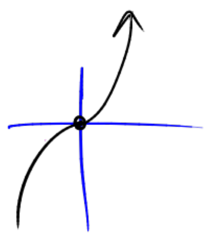
$$\text{If } f(x_1) = f(x_2) \text{ then } x_1 = x_2$$

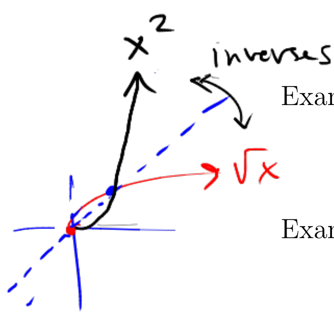
(A) useful when you have some formula.

4. What it means to be 1-1 in a graph: see (A)

still (1-1)
$x(x^2+1)$
$x^3 + x$
$x^3 - x$
↳ not 1-1.

Example: $f(x) = x^3$

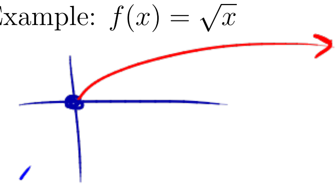




Example: $f(x) = |x|$

NO

Example: $f(x) = \sqrt{x}$



Example: $f(x) = ax + b$ lines always 1-1.

constants

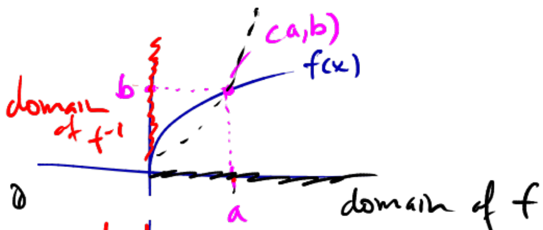
The Inverse of a Function

If f is a 1-1 function with domain A and range B, then its *inverse function* f^{-1} has domain B and range A and s defined by

$$f^{-1}(y) = x \iff f(x) = y$$

1. What it means to be an inverse in words: if f takes x onto y, then $f - inverse$ takes y back onto x.

2. What it means to be an inverse in pictures:



#4

f^{-1} takes 1 back to 0 so

f takes 0 onto 1

3. To find the inverse for specific values: If $f(0) = 1$, $f(2) = 3$, $f(4) = 5$, then If f is 1-1 then

   $f^{-1}(1) =$ 0 $f^{-1}(3) =$ 2 $f^{-1}(5) =$ 4

4. What it means to be 1-1 in a graph:

   - $(a, b) \in graph(f) \iff (b, a) \in graph(f^{-1})$

5. To verify that two functions are inverses: $f(x) = x^2$, $g(x) = \sqrt{x}$.

$$f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x$$

$$g(f(x)) = g(x^2) = \sqrt{x^2} = x$$

check: $f^{-1}(f(x)) = f^{-1}(5x+3) = \frac{1}{5}(5x+3) - \frac{3}{5} = x + \frac{3}{5} - \frac{3}{5} = \underline{\underline{x}}$

How to find the inverse of a 1-1 function

multiply by 5, then add 3

1. Set $y = f(x)$
2. Solve for x
3. Interchange x and y, to get $y = f^{-1}(x)$

multiply by 1/5 then subtract 3/5

Example: Find the inverse of $f(x) = 5x + 3$

$y = 5x+3$ ✓

$y - 3 = 5x$

$\frac{1}{5}y - \frac{3}{5} = \frac{y-3}{5} = x$ ✓

step 3: $\boxed{f^{-1}(x) = \frac{1}{5}x - \frac{3}{5}}$

Example: Find the inverse of $f(x) = \sqrt{x+3}$

$y = \sqrt{x+3}$

$y^2 = x + 3$

$y^2 - 3 = x$

so $y = x^2 - 3$; $f^{-1}(x) = x^2 - 3$

check: $f^{-1}(\sqrt{x+3}) = (\sqrt{x+3})^2 - 3$
$= x + 3 - 3$
$= x$

☺

Example: Find the inverse of $\boxed{f(x) = \frac{\sqrt{x}+6}{7-\sqrt{x}}}$

$y = \frac{\sqrt{x}+6}{7-\sqrt{x}}$ cross mult. $y(7-\sqrt{x}) = \sqrt{x} + 6$

$7y - y\sqrt{x} = \sqrt{x} + 6$

what's goal? solve for x.

$7y - 6 = y\sqrt{x} + \sqrt{x}$

$= \sqrt{x}(y+1)$

so $\frac{7y-6}{y+1} = \sqrt{x}$

now square both sides

$\left(\frac{7y-6}{y+1}\right)^2 = x$

$f^{-1}(x) = \left(\frac{7x-6}{x+1}\right)^2$

let's check a few points:

$f(1) = \frac{\sqrt{1}+6}{7-\sqrt{1}} = \frac{7}{6}$

$f^{-1}\left(\frac{7}{6}\right) = \left(\frac{7\left(\frac{7}{6}\right)-6}{\frac{7}{6}+1}\right)^2$

$= \left(\frac{\frac{49}{6}-\frac{36}{6}}{\frac{7}{6}+\frac{6}{6}}\right)^2 = \left(\frac{\frac{13}{6}}{\frac{13}{6}}\right)^2 = 1^2 = 1$

MA115 :: Section 2.8 :: Inverse Functions

Warm-up 1: If $f(g(x)) = 6(x^7+7)^9$ and $g(x) = x^7+7$, find $f(x)$.

f takes g(x) and … raises it to the 9th, then multiplies by 6 so … f(x) = $6x^9$

Warm-up 2: If $f(x) = x+3$ and $g(x) = \frac{1}{x+3}$, find

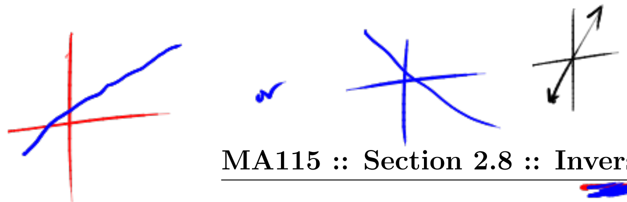
$f(g(2)) =$ $(f \circ g)(2)$__________ and $(g \circ g)(x)$__________

$f(g(x)) = f\left(\frac{1}{x+3}\right) = \boxed{\frac{1}{x+3} + 3} = \frac{1}{x+3} + \frac{3(x+3)}{(x+3)} = \frac{3x+10}{x+3}$, now plug $x=2$

$f(g(2)) = \frac{16}{5} = 3.2$

$g(g(x)) = g\left(\frac{1}{x+3}\right) = \frac{1}{\boxed{\left(\frac{1}{x+3}\right) + 3}} = \frac{1}{\left(\frac{3x+10}{x+3}\right)} = \boxed{\frac{x+3}{3x+10}}$

Ex.1 $f(x) = 2x$. The inverse function $f^{-1}(x)$ ↳ doesn't mean $\frac{1}{f}$



$f^{-1}(f(x)) = x$

f^{-1} undoes f. $f^{-1}(x) = \frac{1}{2}x$

or

$f^{-1}(f(x)) = \frac{1}{2}(2x) = x$.

MA115 :: Section 2.8 :: Inverse Functions

For **certain** functions, there is an equal and opposite function, an *inverse function*. This function "undoes" whatever the function has done. Functions that have inverses are called *one-to-one*.

1. What it means to be 1-1 in words: *never take on the same value twice*

2. What it means to be 1-1 in pictures:

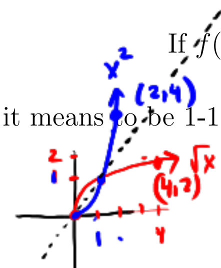
3. What it means to be 1-1 in math:

$$\text{If } f(x_1) = f(x_2) \text{ then } x_1 = x_2$$

1-1 not 1-1

$x^3 \longrightarrow x^3 - x$

4. What it means to be 1-1 in a graph: f & f^{-1} are symmetric about line $y=x$.



are these 1-1?

Yes if graph satisfies HORIZONTAL LINE TEST

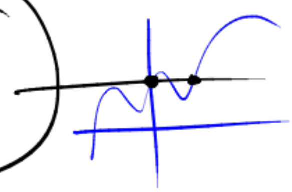
Example: $f(x) = x^3$

1-1

Example: $f(x) = |x|$

not 1-1

not 1-1

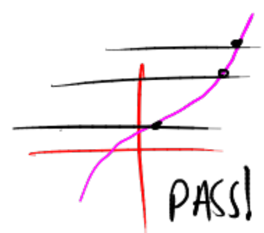


FAILS!

Example: $f(x) = \sqrt{x}$

yes

1-1



PASS!

constant constant

Example: $f(x) = ax + b$

yes, lines are 1-1

(assuming $a \neq 0$)

The Inverse of a Function

If f is a 1-1 function with domain A and range B, then its *inverse function* f^{-1} has domain B and range A and s defined by

$$f^{-1}(y) = x \iff f(x) = y$$

$f : x \longrightarrow y$

$x \longleftarrow y : f^{-1}$

1. What it means to be an inverse in words: if f takes x onto y, then $f - inverse$ takes y back onto x.

2. What it means to be an inverse in pictures:

f is 1-1

3. To find the inverse for specific values: If $f(0) = 1, f(2) = 3, f(4) = 5$, then

   $f^{-1}(1) =$ 0 $f^{-1}(3) =$ 2 $f^{-1}(5) =$ 4

4. What it means to be 1-1 in a graph:

$$(a, b) \in graph(f) \iff (b, a) \in graph(f^{-1})$$

5. To verify that two functions are inverses: $f(x) = x^2$, $g(x) = \sqrt{x}$.

check: $f \circ g(x) = x$

$f(g(x)) = f(\sqrt{x})$

$= (\sqrt{x})^2 = x$

$g(f(x)) = g(x^2) = \sqrt{x^2} = x$

yes!

verify: $f \circ f^{-1}(x) = x$ ... $f^{-1}(f(x)) = \frac{1}{5}(5x+3) - \frac{3}{5} = x + \frac{3}{5} - \frac{3}{5} = \underline{\underline{x}}$

How to find the inverse of a 1-1 function

f(x): multiply by 5, add 3

$f^{-1}(x)$: mult. by $\frac{1}{5}$ then subtract $\frac{3}{5}$

1. Set $y = f(x)$
2. Solve for x
3. Interchange x and y, to get $\underline{y = f^{-1}(x)}$

Example: Find the inverse of $f(x) = \boxed{5x + 3}$

✓ (1)

$y = 5x + 3$

$y - 3 = 5x$

$\frac{y-3}{5} = x$

✓ (2)

$\boxed{x = \frac{1}{5}y - \frac{3}{5}}$

✓ (3)

$\boxed{y = f^{-1}(x) = \frac{1}{5}\boxed{x} - \frac{3}{5}}$

Example: Find the inverse of $f(x) = \sqrt{x + 3}$

$y = \sqrt{x+3}$

$y^2 = (\sqrt{x+3})^2 = x + 3$

$y^2 - 3 = x$

$\boxed{f^{-1}(x) = y = x^2 - 3}$

Example: Find the inverse of $f(x) = \frac{\sqrt{x} + 6}{7 - \sqrt{x}} = y$

goal: isolate x

cross mult.

$(7 - \sqrt{x})y = \sqrt{x} + 6$

$7y - y\sqrt{x} = \sqrt{x} + 6$

$-6 + y\sqrt{x} \qquad -6 + y\sqrt{x}$

$7y - 6 = \sqrt{x} + y\sqrt{x} = \sqrt{x}(1 + y)$

$\frac{7y - 6}{1 + y} = \sqrt{x}$ square both sides

$x = \left(\frac{7y-6}{1+y}\right)^2$

$f^{-1}(x) = y = \boxed{\left(\frac{7x-6}{1+x}\right)^2}$

check

$\frac{\left(\frac{7x-6}{1+x}\right) + 6\left(\frac{1+x}{1+x}\right)}{7\left(\frac{1+x}{1+x}\right) - \frac{7x-6}{1+x}} = x$