$x^2 - 13x + 40$ = $f(x)$

find:
- vertex
- maximum value of (function) expression

$\left(\frac{13}{2}\right)^2 - 13\left(\frac{13}{2}\right) + 40 =$

$\frac{169}{4} - \frac{169}{2}\cdot\frac{2}{2} + 40$

$= -\frac{169}{4} + \frac{40}{4}\cdot 4$

$= -\frac{9}{2}$

- some solution:

1. vertex $= \left(-\frac{b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$b = -13$, $a = 1$

$= \left(-\frac{(-13)}{2(1)} = \frac{13}{2}, f\left(\frac{13}{2}\right)\right)$

$= \left(\frac{13}{2}, -\frac{9}{2}\right)$

2. comple $\square$

$(x^2 - 13x) + 40 = x^2 - 13x + \left(\frac{13}{2}\right)^2 + 40 - \left(\frac{13}{2}\right)^2$

$\frac{160}{4}$ $\quad -\frac{169}{4}$

$0 = \left(x - \frac{13}{2}\right)^2 - \frac{9}{2}$

$\sqrt{\frac{9}{2}} = \sqrt{\left(x - \frac{13}{2}\right)}$

$\pm\sqrt{\frac{9}{2}} = x - \frac{13}{2}$

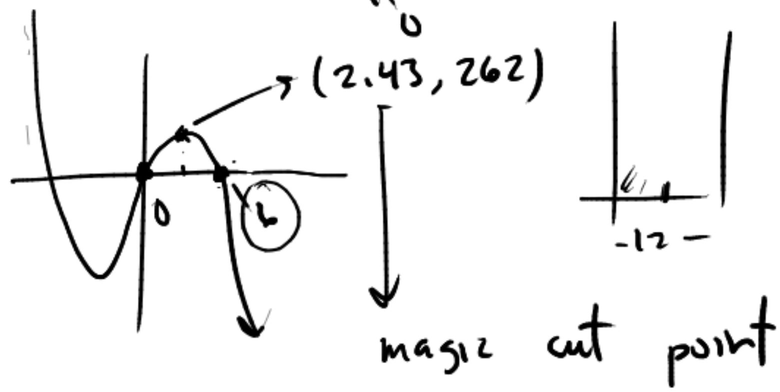
so $x = \frac{13}{2} \pm \sqrt{\frac{9}{2}}$

$= \frac{13}{2} \pm \frac{3}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

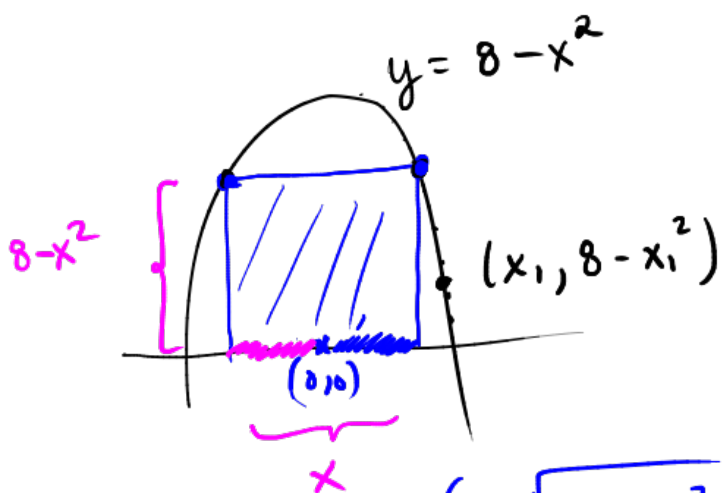
$= \frac{13}{2} \pm \frac{3\sqrt{2}}{2} = \boxed{\frac{13 \pm 3\sqrt{2}}{2}}$

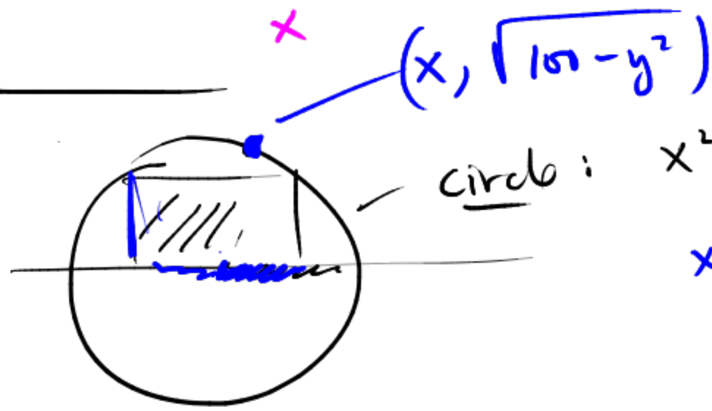
$V(x) = x(12-2x)(20-2x)$

degree 3
negative leading term



$A = (x)(8-x^2)$





circle: $x^2 + y^2 = r^2$

$x^2 + y^2 = 100$

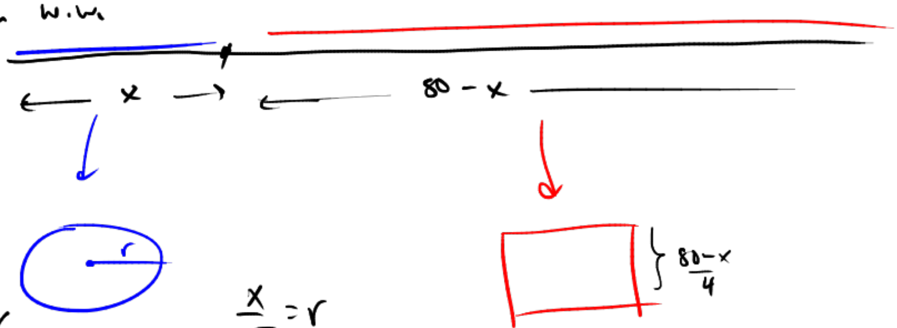
$x^2 = 100 - y^2$

$x = \sqrt{100-y^2}$

$A = (2x)\sqrt{100-y^2}$

#18 on W.W.

80 cm



$\frac{x}{2\pi} = r$

Circumference = $\boxed{x = 2\pi r}$

Need to relate r to x.

$\boxed{A = \pi r^2 + \left(\frac{80-x}{4}\right)^2}$

Area Circle — Area of Square

vertex

(*) $x = C = 2\pi r$

$\boxed{\frac{x}{2\pi} = r}$

(*) $= \frac{\pi x^2}{4\pi^2} + \frac{80^2 - 160x + x^2}{16} = \underbrace{\left(\frac{1}{4\pi} + \frac{1}{16}\right)}_{a} x^2 \underbrace{- \left(\frac{160}{16}\right)}_{b} x + \frac{80^2}{16}$

$= -10$

vertex ⟷ (spot to cut s.t. A is a minimum, sum of areas)

‖

where to cut = $\frac{-b}{2a}$ = circumference

a =

Wed: 2.7 - composing functions
Thur: 2.8 - inverse functions
Fri: 3.1 - deeper dicussion of polys
Mon & Wed: More Chapter 3.2/3.3
Thurs: review for exam
Fri: exam 2

WeBWork: (2.6)

height = $y = 80t - 16t^2$

what's max height? = vertex

$= \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

where $f(t) = 80t - 16t^2$

$= -16t^2 + 80t$

$\frac{-b}{2(a)} = \frac{-80}{2(-16)} = \frac{10}{4} = \frac{5}{2}$

$f\left(\frac{5}{2}\right) = 80\left(\frac{5}{2}\right) - 16\left(\frac{5}{2}\right)^2$

$= 200 - 100 = 100$

$-16 \cdot \frac{25}{4}$

$= -4 \cdot 25$

$= -100$

MA115 :: Section 2.7 :: Combining Functions

Four easy way to make new functions from old ones

1. Add: $(f+g)(x) = f(x) + g(x)$, ex: $\boxed{\begin{array}{l} f(x) = x^3 \\ g(x) = -x+1 \end{array}}$ $(f+g)(x) = x^3 - x + 1$
2. Subtract: $(f-g)(x) = f(x) - g(x)$ ex: $(f-g)(x) = x^3 + x - 1$
3. Multiply: $(fg)(x) = f(x)g(x)$ ex: $(fg)(x) = (x^3)(-x+1)$
4. Divide: $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$ ex: $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^3}{-x+1}$

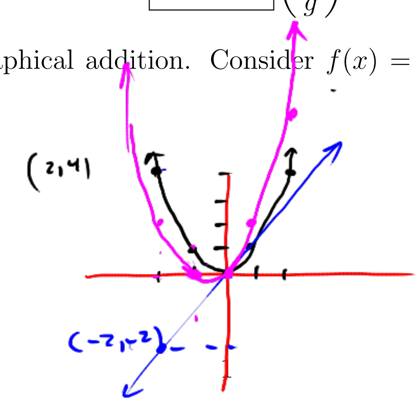
The domain of the new function is the intersection of the domains of the original functions, minus any points that make the new function undefined.

If $f(x) = x^2$, $g(x) = x - 2$, $h(x) = \dfrac{1}{x}$, $H(x) = \dfrac{1}{x^2}$, $F(x) = 1$ $G(x) = \sqrt{x}$ compute:

Domain			Domain
$\mathbb{R}$	$(f+g)(x) = x^2 + x - 2$	$(f-h)(x) = x^2 - \frac{1}{x}$	$\mathbb{R} - \{0\}$
$\mathbb{R}$	$(f+F)(x) = x^2 + 1$	$(gF)(x) = x - 2$	$\mathbb{R}$
$\mathbb{R} - \{0\}$	$\left(\dfrac{g}{f}\right)(x) = \dfrac{x-2}{x^2}$	$(fH)(x) = (x^2)\frac{1}{x^2} = \boxed{1}$	$\mathbb{R}$
$\mathbb{R}$	$\left(\dfrac{g}{h}\right)(x) = \dfrac{x-2}{\frac{1}{x}} = x(x-2)$ $\boxed{= x^2 - 2x}$	$\left(\dfrac{G}{g}\right)(x) = \dfrac{\sqrt{x}}{x-2}$	$\mathbb{R} - \{2\}$

We can use graphical addition. Consider $f(x) = x^2$ and $g(x) = x$ and $h(x) = 1$.

$(f+g)(x) = x^2 + x$



MA115 :: Section 2.7 :: Combining Functions

The output from one function becomes input into another

The most important way to combine functions is by **composition of functions**.

You basically apply each function rule, one after the other.

input

output goes into $f(x)$ to compute

For example, if $f(x) = x^2$ and $g(x) = 2x + 1$, we write

$f(2x+1) = (2x+1)^2 = 4x^2+4x+1$

f composed with g …. (or) f of g of x

$$h(x) = f(g(x)) = (f \circ g)(x)$$

So you first applied the rule g, then applied the rule f. Notation:

$$(f \circ g)(x) = f(g(x))$$

If $f(x) = x^2$, $g(x) = x - 2$, $h(x) = \frac{1}{x}$, $H(x) = \frac{1}{x^2}$, $F(x) = 1$ $G(x) = \sqrt{x}$ compute:

$(f \circ g)(x) =$ $f(g(x)) = f(x-2) = (x-2)^2 = x^2 - 4x + 2$

$(f \circ h)(x) =$ $f(\frac{1}{x}) = \frac{1}{x^2}$

$(H \circ f)(x) =$ $H(f(x)) = H(x^2) = \frac{1}{x^4}$

$(f \circ H))(x) =$ $f(\frac{1}{x^2}) = \frac{1}{x^4}$

$(h \circ H)(x) =$ $h(H(x)) = h(\frac{1}{x^2}) = \frac{1}{\frac{1}{x^2}} = x^2$

$(H \circ g \circ f))(x) =$ $H(g(x^2)) = H(x^2-2) = \frac{1}{(x^2-2)^2}$

Also:

$\boxed{h \circ h(x) = x}$

$g \circ g \circ g(x) = x - 6$

$\boxed{(f \circ G)(x) = f(\sqrt{x}) = x}$

Application: Imagine x is the retail price of a iPhone, you have a \$50 coupon from Apple and RadioShack offers a 20% discount on all phones. Give functions that model the purchase price of the iPhone after each of the discounts, as well as if both discounts are allowed.

$C_1(x)$ = cost of iPhone after \$50 coupon $= x - 50$

$C_2(x)$ = " " " " 20% discount $= x - .2x = .8x$

$C_3(x)$ = cost if you first take 20% off then apply coupon: $C_1 C_2(x) = C_1(C_2(x)) = C_1(.8x) = \boxed{.8x - 50}$

$C_4(x)$ = cost if you apply coupon then take 20% off:

$C_2 \circ C_1(x) = C_2(x-50) = .8(x-50) = \boxed{.8x - 40}$

$$f(x) = \frac{1}{1 - \frac{1}{x}}$$

$$f \circ f(x) = \frac{1}{1 - \frac{1}{\frac{1}{1-\frac{1}{x}}}} \qquad \frac{1}{1-\frac{1}{x}} = \frac{1}{\frac{x}{x} - \frac{1}{x}} = \frac{1}{\frac{x-1}{x}} = \frac{x}{x-1}$$

$$= \frac{1}{1 - \frac{x-1}{x}}$$ → this is flipped

$$= \frac{1}{\frac{x}{x} - \frac{(x-1)}{x}} = \frac{1}{\frac{x-x+1}{x}} = \frac{1}{\frac{1}{x}} = x$$

#9 (Last Week's worksheet)

what are the solutions to : $ax^2+bx+c=0$?

$a\neq 0$, a, b, c constants.

complete the square
+ alotta algebra

$$(ax^2+bx)+c=0$$

$$a\left(x^2+\frac{b}{a}x\right) + c = a\underbrace{\left(x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2\right)}\ +\overbrace{c-\left(\frac{b}{2a}\right)^2 a}^{c-\frac{b^2}{4a}}$$

insert

compensate

$$= a\left(x+\frac{b}{2a}\right)^2 + c - \frac{b^2}{4a} = 0$$

$$= \frac{1}{a}\,a\left(x+\frac{b}{2a}\right)^2 = \frac{b^2}{4a} - \frac{c\cdot 4a}{4a} = \left(\frac{b^2-4ac}{4a}\right)\frac{1}{a}$$

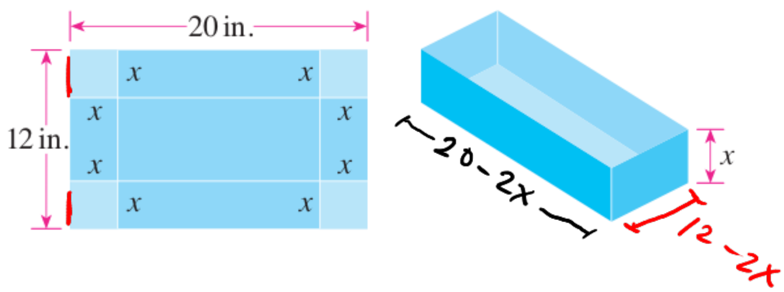
$$= x = -\frac{b}{2a} \pm \sqrt{\frac{b^2-4ac}{4a^2}} = \boxed{\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

Volume of a Box A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 12 in. by 20 in. by cutting out equal squares of side x at each corner and then folding up the sides (see the figure).

✓ **(a)** Find a function that models the volume of the box.

(b) Find the values of x for which the volume is greater than 200 in^3. $[1.17, 3.9] \to V \geq 200$

(c) Find the largest volume that such a box can have. $(262.7$ in^3, occurring at $x = 2.43)$



(a) $V = x(20-2x)(12-2x)$

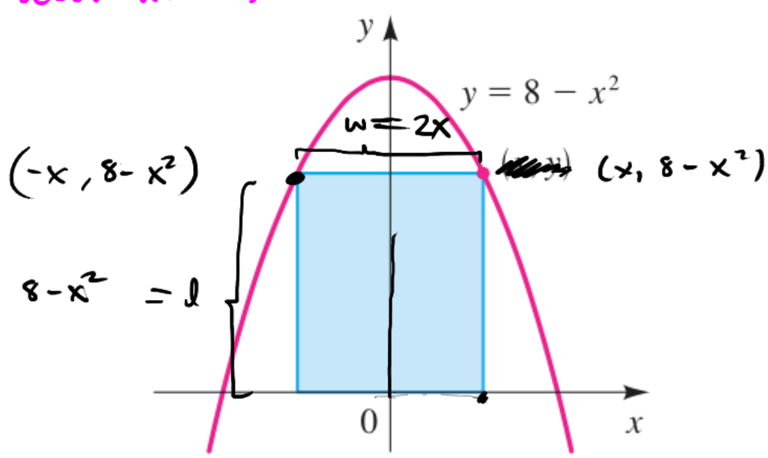
degree 3 ... completing square is of no help to solve max/min problems.

Inscribed Rectangle Find the dimensions that give the largest area for the rectangle shown in the figure. Its base is on the x-axis and its other two vertices are above the x-axis, lying on the parabola $y = 8 - x^2$.

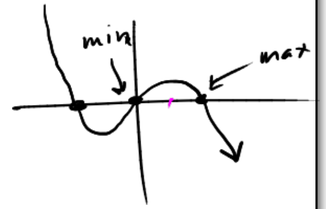
degree 3 ... use graph ....

$A = l \cdot w$
$= (8-x^2)(2x)$
$=$ " "

largest Area: occur when $x = 1.63$.



zeros of this are $x = 0, x = \pm 2\sqrt{2}$

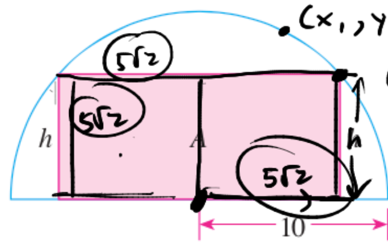


$A = l \cdot w = (2x)\cdot h = (2\sqrt{100-h^2})h = 2h\sqrt{100-h^2}$

$x^2+y^2=100$

Area A rectangle is inscribed in a semicircle of radius 10, as shown in the figure. Find a function that models the area A of the rectangle in terms of its height h.

$h = 5\sqrt{2}$



$(x_1, y_1) \Rightarrow x_1^2 + y_1^2 = 100$

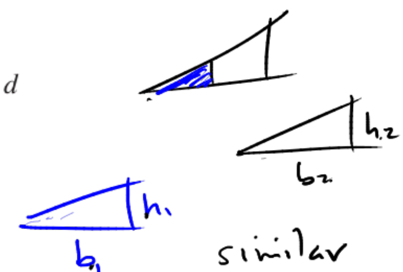
$(x, h) \Rightarrow x^2 + h^2 = 100$

$x^2 = 100 - h^2$

$x = \sqrt{100-h^2}$

$(\sqrt{100-h^2}, h)$

Length A woman 5 ft tall is standing near a street lamp that is 12 ft tall, as shown in the figure. Find a function that models the length L of her shadow in terms of her distance d from the base of the lamp.



similar Δ's

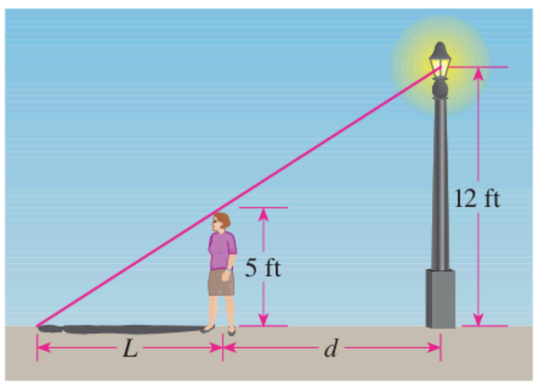
$\frac{b_1}{h_1} = \frac{b_2}{h_2}$

$\frac{5}{L} = \frac{12}{L+d}$

$5L + 5d = 12L$

$5d = 7L$

$\frac{5}{7}d = L$



Light from a Window A Norman window has the shape of a rectangle surmounted by a semicircle, as shown in the figure. A Norman window with perimeter 30 ft is to be constructed.

(a) Find a function that models the area of the window.

(b) Find the dimensions of the window that admits the greatest amount of light.

$\boxed{x = -\frac{7.5}{\left(-\frac{2+\pi}{2}\right)} = \frac{15}{2+\pi} \text{ is the width; the other dim's can easily be found.}}$

half a circle —

Area $= \frac{1}{2}\pi r^2$. Here $r = \frac{x}{2}$ so

$= \frac{1}{2}\frac{\pi x^2}{4} = \boxed{\frac{\pi x^2}{8}}$

$A_2 = x \cdot y$

Here:

$30 = 2y + x + \frac{1}{2}C$

where $C = 2\pi r = 2\pi \frac{x}{2} = \pi x$

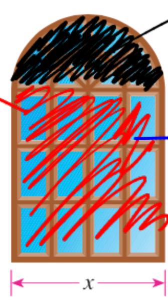
so

$30 = 2y + x + \frac{\pi x}{2}$

thus (mult. by 2) ...

$60 = 4y + (2+\pi)x$

$y = \frac{1}{4}(60 - (2+\pi)x) = 7.5 - \frac{x}{2} - \frac{\pi x}{4}$



Now $A_2 = \frac{1}{4}(60x - (2+\pi)x^2)$

So $A = \underbrace{7.5x - \frac{x^2}{2} - \frac{\pi x^2}{4}}_{xy} + \frac{\pi x^2}{8}$

$a = -\frac{(2+\pi)}{4}$

$b = 7.5$

$= -\frac{\pi x^2}{4} - \frac{2x^2}{4} + 7.5x = \boxed{-\frac{(2+\pi)x^2}{4} + 7.5x}$