Practice Exam 2 :: Math 115

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1. **Common Evaluations**
   Assume $h \neq 0$ and $f(x) = 2x^2 - x$. Evaluate

$$\frac{f(a+h) - f(a)}{h}.$$

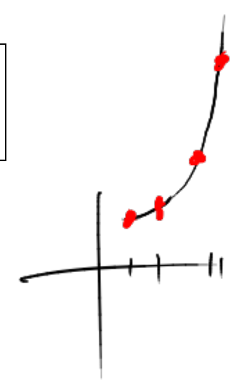
$$\frac{2(a+h)^2 - (a+h) - (2a^2 - a)}{h}$$

$$(2a^2 + 4ah + 2h^2 - a - h - 2a^2 + a) \cdot 1/h$$

$$(4ah - h + 2h^2) \cdot 1/h = \boxed{4a - 1 + 2h}$$

(b) now let h=0, and substitute into your answer $\Rightarrow \boxed{4a-1}$

2. **Rate of Change**
   Compute the average rate of change of the function



$$f(x) = x^3 - \frac{1}{x}$$

on the interval $[1, 5]$.

$$\frac{f(5) - f(1)}{5-1} = \frac{\left(5^3 - \frac{1}{5}\right) - \left(1^3 - \frac{1}{1}\right)}{4} = \frac{125 - \frac{1}{5}}{4} = \frac{125 - .2}{4} = \frac{124.8}{4} \approx 31.2$$

.2

(b) repeat for [10, 11]

$$\frac{f(11) - f(10)}{11-10} = \frac{\left(11^3 - \frac{1}{11}\right) - \left(10^3 - \frac{1}{10}\right)}{1} \approx 11^3 - 10^3 = (10+1)^3 - 10^3$$

$$= 10^3 + 3 \cdot 10^2 \cdot 1 + 3 \cdot 10 \cdot 1^2 + 1 - 10^3$$

$$= 331$$

compare

1
1 1
1 2 1
1 3 3 1

3. **The Domain of Functions**
Find the domain of each of the functions

$$\frac{\sqrt{(x+3)}}{2x^2+x-1}$$

$x+3 \geq 0$
$x \geq -3$

Throw out these solutions

$\underbrace{2x^2+x-1}=0$

$(2x-1)(x+1)=0$

$x=\frac{1}{2}, \quad x=-1$

Domain

$x \geq -3, \; x \neq \frac{1}{2}, x \neq -1$

or

$[-3,-1) \cup (-1,\frac{1}{2}) \cup (\frac{1}{2},\infty)$

(b) $\dfrac{1}{1+\dfrac{1}{1-\boxed{x}}}$

$\boxed{\begin{array}{c}\text{Domain}\\ \mathbb{R}-\{1,2\}.\end{array}}$

start: see $x \neq 1$.

also $\frac{(1-x)}{(1-x)} 1 + \frac{1}{1-x} = 0$

$\frac{1-x+1}{1-x}=0$ the sols are: $2-x=0$, $x=2$ throw out

$$\frac{1}{\sqrt{(x+3)}}$$

strictly greater than 0

$x+3>0$

$\boxed{x>-3}$

4. Graphs of Functions

Graph the each of the following polynomials, showing clearly all $x-$ and $y-$intercepts.

I want to see:

vertex: (2,1)

quadratic, negative leading term

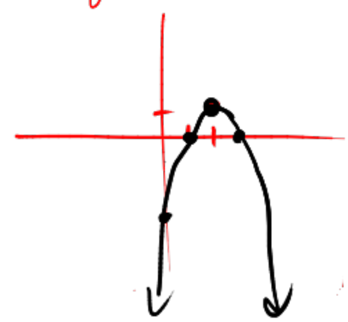
$a(x-h)^2+k$

standard form

$f(x) = -(x-2)^2 + 1$

h k

given



x-int: $y=0 = \boxed{-(x-2)^2+1} \Rightarrow (x-2)^2 = 1$

$x-2 = \pm 1$

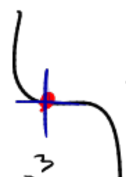
$x = 2 \pm 1 \Rightarrow x = 3$ or $x = 1$

y-int: $x = 0$

$\Rightarrow y = -3$

$f(x) = -(x-2)^3 + 1$

$x=0$
$y=9$



= shape

$y=0$

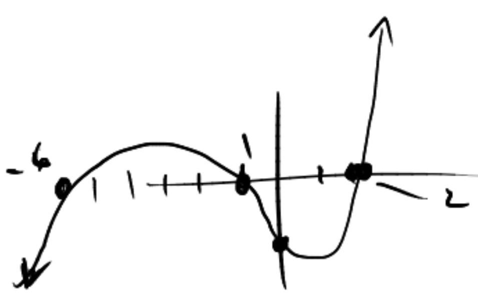
$0 = -(x-2)^3 + 1$

$(x-2)^3 = 1$

$(x-2) = 1$

$x = 3$

$f(x) = (x-2)(x+1)(x+6)$

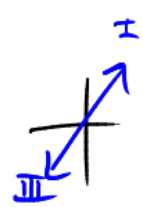


$x = 0 \Rightarrow y = -12$

degree 3

3 is odd

leading coef is positive



similar to $y = x^1$

+

5. Consider the following rational functions:

$$r(x) = \frac{2x-1}{x^2-x-2}, \quad s(x) = \frac{x^3+27}{x^2+4}, \quad f(x) = \frac{x^3-9x}{x+2}, \quad g(x) = \frac{x^2+x-6}{x^2-25}$$

(a) Which of these rational functions has a horizontal asymptote?

(b) Which of these rational functions has a slant asymptote?

(c) Which of these rational functions has no vertical asymptote?

(d) Graph $y = g(x)$ showing clearly any asymptotes.

Solution to 6:

set $\boxed{h(x) = f(x) \cdot g(x)}$. $h(x)$ is even if $h(-x) = h(x)$.

plug in $-x$: $h(-x) = f(-x) \cdot g(-x)$

" $[-f(x)] \cdot [-g(x)] = f(x) \cdot g(x) = h(x)$

use this

6. **Even and Odd Functions**
What is the definition of an even function? What is the definition of an odd function?

odd: $f(-x) = -f(x)$

even: $f(-x) = f(x)$

b) if f(x) and g(x) are both ODD, is f(x) times g(x) odd, even or neither?

7. **Combining Functions**
Let $f(x) = 2x$ and $g(x) = x^2 + 1$. Compute :

all odd exponents = f(x) odd

$f(g(x)) =$ $f(x^2+1) = 2x^2+2$

$g(f(x)) =$ $g(2x) = (2x)^2 + 1 = 4x^2 + 1$

Also, determine whether the following are odd, even or neither.

$f(x)$ odd (only odd exponents)

$g(x)$ even only even exponents

$f(g(x))$ even

$g(f(x))$ even

$h(x) = x^2 + x^3$

$h(-x) = (-x)^2 + (-x)^3$

$= x^2 - x^3 \neq h(x)$

$\neq -h(x)$

$\Rightarrow$ neither

8. Find the inverse of the function

goal isolate x then switch x, y

$$y = f(x) = \frac{x-1}{x+2}$$

$y(x+2) = x - 1$

distribute

$yx + 2y = x - 1$

$yx - x = -2y - 1$

$x(y-1)$

$x = -\frac{2y-1}{y-1}$

switch

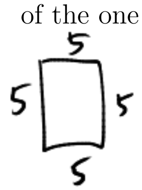
$$\boxed{y = f^{-1}(x) = \frac{-2x-1}{x-1}}$$

(b) check/verify your answer

$f(f^{-1}(x)) = x$ show this.

$$\frac{\frac{-2x-1}{x-1} - 1\left(\frac{x-1}{x-1}\right)}{\frac{-2x-1}{x-1} + 2\left(\frac{x-1}{x-1}\right)} = \frac{\frac{-2x-1-x+1}{\cancel{x-1}}}{\frac{-2x-1+2x-2}{\cancel{x-1}}} = \frac{-3x}{-3} = \boxed{x}$$

9. Among all rectangles that have a perimeter of 20 ft, find the dimensions of the one with the largest area.



, duh.

$P = 20 = 2l + 2w$

$20 - 2l = 2w$

$10 - l = w$

Area $= l \cdot w = l(10 - l)$

$= -l^2 + 10l$

vertex $= (h, k)$

$h = \frac{-b}{2a} = \frac{-10}{2(-1)} = 5$

$y = x$

x^3

x^5

x^7

$y = -x$

$-x^3$

$-x^7$

x^{10}

$-x^8$

$f(x) = \frac{x-1}{x+2} \Rightarrow y = \frac{x-1}{x+2} \Rightarrow y(x+2) = x-1 \Rightarrow yx + 2y = x-1$

goal: isolate x

$$\begin{array}{rcl} yx + 2y & = & x - 1 \\ -x & & -x \\ \hline yx - x + 2y & = & -1 \\ -2y & & -2y \\ \hline yx - x & = & -1-2y \end{array}$$

$$x(y-1) = -1-2y$$

$$f^{-1}(x) = \frac{-1-2x}{x-1} \Leftarrow x = \frac{-1-2y}{y-1}$$