$= \frac{8xh + 4h^2 - 3h}{h} = \boxed{8x + 4h - 3}$

$h \to 0$

Name: ______________________ :: Exam 2 :: Math 115 :: Fall 2015

1. Assume $h \neq 0$ and $f(x) = 4x^2 - 3x + 2$. Compute $\dfrac{f(x+h) - f(x)}{h}$

$\dfrac{4(x+h)^2 - 3(x+h) + 2 - (4x^2 - 3x + 2)}{h} = \dfrac{4x^2 + 8xh + 4h^2 - 3x - 3h + 2 - 4x^2 + 3x - 2}{h}$

What do you get when you substitute $h = 0$ into your answer? $8x - 3$

2. Compute the average rates of change of the following functions on the given intervals

$\dfrac{f(b) - f(a)}{b - a}$

$f(x) = \dfrac{1}{x}$ on the interval $\left[\dfrac{1}{10}, \dfrac{1}{5}\right]$ -50

$a = \frac{1}{10}$, $b = \frac{1}{5}$

$\dfrac{5 - 10}{\frac{2}{10} - \frac{1}{10}} = \dfrac{-5}{\frac{1}{10}} \to \boxed{-5 \cdot \left(\frac{10}{1}\right)}$

$f(x) = \dfrac{1}{x}$ on the interval $[5, 10]$ $-\frac{1}{50}$

$\dfrac{\frac{1}{10} - \frac{2}{10}}{10 - 5} = \dfrac{-\frac{1}{10}}{5} = -\frac{1}{10} \cdot \frac{1}{5}$

$g(x) = x^2$ on the interval $\left[\dfrac{1}{10}, \dfrac{1}{5}\right]$ $3/10 = .3$

$\dfrac{\frac{4}{100} - \frac{1}{100}}{\frac{1}{5} - \frac{1}{10}} = \dfrac{3/100}{\frac{1}{10}} =$

$g(x) = x^2$ on the interval $[5, 10]$ 15

$\dfrac{100 - 25}{10 - 5} = \dfrac{75}{5} =$

Discussion: How do these average rates of change relate to the graphs of $f(x) = \frac{1}{x}$ and $g(x) = x^2$?

Slope, needed to be mentioned.

3. What is the definition of an **even** function?

$$f(-x) = f(x)$$

What is the definition of an **odd** function?

$$f(-x) = -f(x)$$

4. Justify the following statements

If $F(x)$ and $G(x)$ are both odd functions then $F(x) + G(x)$ is odd

$$F(-x) + G(-x) = -F(x) - G(x) = -(F(x) + G(x))$$

Bare minimum

negative in

negative out

If $F(x)$ and $G(x)$ are both odd functions then $F(x) * G(x)$ is even

$$F(-x) \cdot G(-x) = (-F(x)) \cdot (-G(x)) = F(x)G(x)$$

5. Determine whether the following are odd, even or neither.

$x^4 - 3x^2 + 5$ even

$\frac{1}{x}$ odd

6. Find the domain of each of the functions

$$f(x) = \frac{\sqrt{x-4}}{x^2+x-2}$$

$x - 4 \geq 0$

$x \geq 4$

— THROW OUT —

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2$

$x = 1$

$x \geq 4$ & $x \neq -2$ & $x \neq 1$

$$g(x) = \frac{1}{x - \frac{1}{1-x}}$$

start: $x \neq 1$.

next:

$\left(\frac{1-x}{1-x}\right) x - \frac{1}{1-x} \neq 0$

$\frac{x - x^2 - 1}{1-x} = 0 \iff -x^2 + x - 1 = 0$

$\iff x^2 - x + 1 = 0$

only complex solutions

$x = \frac{-(-1) \pm \sqrt{1-4}}{2}$

$= \boxed{\frac{1 \pm \sqrt{3}\, i}{2}}$

$x \neq 1$

7. Find the inverse of the function

$$f(x) = 6x^3 + 6$$

$y = 6x^3 + 6$

$\frac{y-6}{6} = \frac{6x^3}{6}$

$\frac{y-6}{6} = x^3 \Rightarrow \sqrt[3]{\frac{y-6}{6}} = x$

$y = \sqrt[3]{\frac{y-6}{6}}$

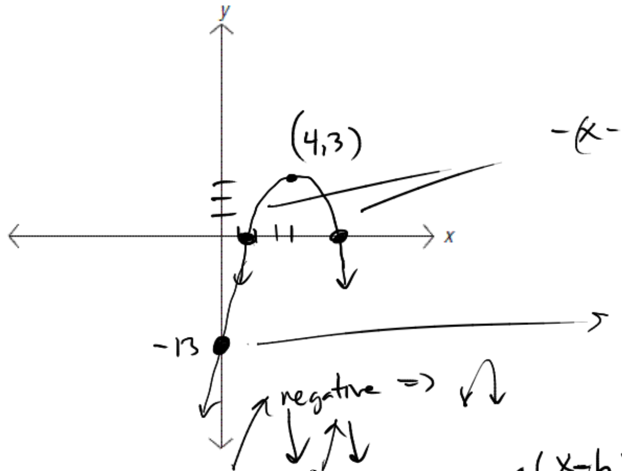
Verify that what you found is indeed the inverse of f(x).

plug into $f(x) = 6x^3 + 6$ ✓

$6\left(\sqrt[3]{\frac{y-6}{6}}\right)^3 + 6$

$6\left(\frac{y-6}{6}\right) + 6$

$y - 6 + 6 = y$



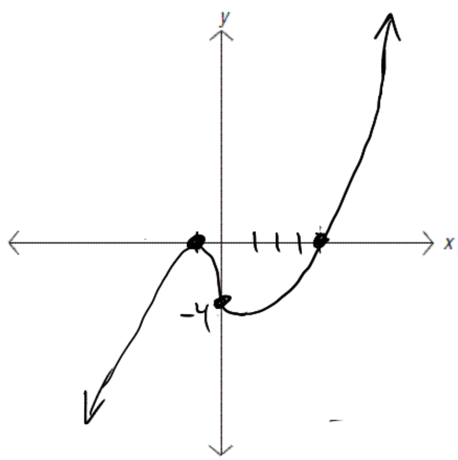
(a) $f(x) = -(x-4)^2 + 3$

$-(x-4)^2 + 3 = 0 \Rightarrow (x-4)^2 = 3$

$x = 4 \pm \sqrt{3}$

$x = 0 \Rightarrow -(0-4)^2 + 3 = -13$

$a(x-h)^2 + k$



(b) $f(x) = (x+1)^2(x-4)$

DEGREE 3, Positive leading term

8. Sketch a graph of the functions above, clearly showing clearly all $x-$ and $y-$intercepts, and the vetex, if appropriate.

Long Division of Polynomials.

$$\frac{5x^3 - 2x^2 + x - 3}{x^2+1} = 5x - 2 + \left(\frac{-4x-1}{x^2+1}\right)$$

If the remainder is 0, then the divisor is a factor.

$$
\begin{array}{r l}
 & 5x - 2 \quad = \text{quotient} \\
x^2+1 \,\big) & 5x^3 - 2x^2 + x - 3 \\
 & -(5x^3 + 5x) \\
 & -2x^2 - 4x - 3 \\
 & -(-2x^2 - 2) \\
 & \boxed{-4x - 1}
\end{array}
$$

$\boxed{-4x-1} \longrightarrow$ degree $= 1$ < degree (x^2+1)

smaller degree

$\Rightarrow$ stop.

Remainder

$$
\begin{array}{r l}
 & x - 3 \\
x - 4 \,\big) & x^2 - 7x + 12 \\
 & -(x^2 - 4x) \\
 & -3x + 12 \\
 & -(-3x + 12) \\
 & 0
\end{array}
$$

$0 \longrightarrow$ remainder $\Rightarrow$ $x-4$ is a factor

$f(x) = x^4 - 1$.

We know $f(1) = 0$

$x = 1$ is a zero.

$\Rightarrow$ $x-1$ is a factor

what's the quotient?

$x^4 - 1 = (x-1)(x^3 + x^2 + x + 1)$

$$
\begin{array}{r l}
 & x^3 + x^2 + x + 1 \\
x - 1 \,\big) & x^4 \qquad\qquad -1 \\
 & -(x^4 - x^3) \\
 & x^3 \qquad\qquad -1 \\
 & -(x^3 - x^2) \\
 & x^2 \qquad\qquad -1 \\
 & -(x^2 - x) \\
 & x - 1 \\
 & x - 1 \\
 & 0
\end{array}
$$

$f(2) = \frac{2-1}{2+2} = \frac{1}{4}$

9. Let $f(x) = \dfrac{x-1}{x+2}$ and $g(x) = \dfrac{1}{x}$.

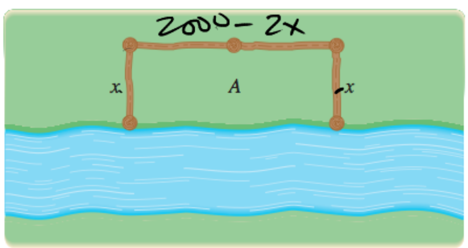
$\dfrac{1}{\left(\frac{1}{4}\right)} = 4$

Compute $g(f(2)) =$ 4

Give a simplified expression for $f(g(x)) =$

$$\frac{\frac{1}{x} - 1\left(\frac{x}{x}\right)}{\frac{1}{x} + 2\left(\frac{x}{x}\right)} = \frac{\frac{1-x}{\cancel{x}}}{\frac{1+2x}{\cancel{x}}} = \frac{1-x}{1+2x}$$

10. Seeds-And-Spores is a small family farm that is located on the banks of the Chocolay River just south of Marquette, MI. Suppose Jeff, one of the farmers, has 2000 ft of fencing that he wants to use to fence off a rectangular field that borders the river. **What are the dimensions of the field of largest area that he can fence?**



500

$A = l \cdot w$

$= (2000 - 2x)x$

$= 2000x - 2x^2$

$-\dfrac{b}{2a} = \dfrac{-2000}{2(-2)} = 500$

11. Fill in the blank: I tell my friends this class is ______

$$\frac{4(x+h)^2-3(x+h)+2-(4x^2-3x+2\qquad)}{h}$$

Name: ______________________________ :: Exam 2 :: Math 115 :: Fall 2015

1. Assume $h \neq 0$ and $f(x) = 4x^2 - 3x + 2$. Compute $\dfrac{f(x+h) - f(x)}{h}$

$$\left(\cancel{4x^2} + 8xh + 4h^2 - \cancel{3x} - 3h + \cancel{2} \;\; \cancel{-4x^2} \;\; \cancel{+3x} \;\; \cancel{-2}\right)/h = \frac{8xh+4h^2-3h}{h} = 8x - 3 + 4h$$

What do you get when you substitute $h = 0$ into your answer? $8x-3$

2. Compute the average rates of change of the following functions on the given intervals

$f(x) = \dfrac{1}{x}$ on the interval $\left[\dfrac{1}{10}, \dfrac{1}{5}\right]$ -50 – big Δy

small Δx

$f(x) = \dfrac{1}{x}$ on the interval $[5, 10]$ $-\frac{1}{50}$ – small Δy

big Δx

$\dfrac{g(b)-g(a)}{b-a}$

$g(x) = x^2$ on the interval $\left[\dfrac{1}{10}, \dfrac{1}{5}\right]$ small Δx $.3$ small Δy

$$\frac{\frac{4}{4}\frac{1}{25} - \frac{1}{100}}{\frac{21}{25} - \frac{1}{10}} = \frac{\frac{3}{100}}{\frac{1}{10}} = \frac{3}{100} \cdot \frac{10}{1} = \frac{3}{10} = .3$$

$g(x) = x^2$ on the interval $[5, 10]$ big Δx 15 big Δy

$$\frac{100-25}{10-5} = \frac{75}{5} = 15$$

Discussion: How do these average rates of change relate to the graphs of $f(x) = \frac{1}{x}$ and $g(x) = x^2$?

Slope

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What is the definition of an **odd** function?

$$f(-x) = -f(x)$$

4. Justify the following statements

If $F(x)$ and $G(x)$ are both odd functions then $F(x) + G(x)$ is odd

Bare Minimum: $F(x) + G(x)$

neg in

neg out

$$\underbrace{F(-x)}_{\text{odd}} + \underbrace{G(-x)}_{\text{odd}} = -F(x) - G(x) = -(F(x) + G(x))$$

If $F(x)$ and $G(x)$ are both odd functions then $F(x) * G(x)$ is even

$$F(-x) \cdot G(-x) = \underbrace{-F(x) \cdot (-G(x))}_{\text{using oddity}} = F(x)G(x)$$

$(-)\cdot(-) = +$

5. Determine whether the following are odd, even or neither.

$x^4 - 3x^2 + 5$ even

$\frac{1}{x} = x^{-1}$ odd

6. Find the domain of each of the functions

$$f(x) = \frac{\sqrt{x-4}}{x^2+x-2}$$

$x-4 \geq 0 \Rightarrow x \geq 4$

$(x+2)(x-1) = 0$

$x = -2, 1$

throw out

$x \geq 4$

$x \geq 4$ & $x \neq -2$, $x \neq 1$

$$g(x) = \frac{1}{\left[x - \frac{1}{1-x}\right]} \neq 0$$

$\boxed{x \neq 1}$

start: $1-x \neq 0$

next. $\left(\frac{1-x}{1-x}\right)x - \frac{1}{1-x} = \frac{x-x^2-1}{1-x} = 0 \iff -x^2+x-1=0$

$x^2-x+1=0$

no real sols.

7. Find the inverse of the function

$$f(x) = 6x^3 + 6$$

$f^{-1}(x) = \left(\frac{x-6}{6}\right)^{1/3}$

$y = 6x^3 + 6$

$\frac{y-6}{6} = \frac{\cancel{6}x^3}{\cancel{6}}$

$\sqrt[3]{\frac{y-6}{6}} = x$

Verify that what you found is indeed the inverse of f(x).

$f(x) = 6x^3 + 6$

$= 6\left(\sqrt[3]{\frac{y-6}{6}}\right)^3 + 6$

$= 6\left(\frac{y-6}{6}\right) + 6 \Rightarrow y-6+6 = y$

$f(2) = \frac{2-1}{2+2} = \frac{1}{4}$

9. Let $f(x) = \dfrac{x-1}{x+2}$ and $g(x) = \dfrac{1}{x}$.

Compute $g(f(2)) =$ 4

$\frac{1}{4}$

Give a simplified expression for $f(g(x)) =$

$$\frac{\frac{1}{x} - 1\left(\frac{x}{x}\right)}{\frac{1}{x} + 2\left(\frac{x}{x}\right)} = \frac{\frac{1-x}{\cancel{x}}}{\frac{1+2x}{\cancel{x}}} = \frac{1-x}{1+2x}$$

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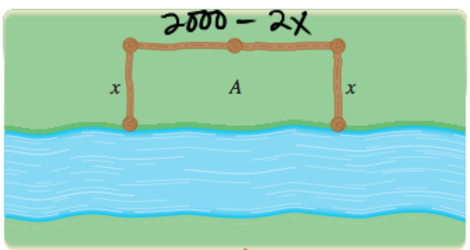


$\frac{2000}{4} = 500$

$$A = l \cdot w$$
$$= (2000 - 2x)\,x$$
$$= 2000x - 2x^2$$

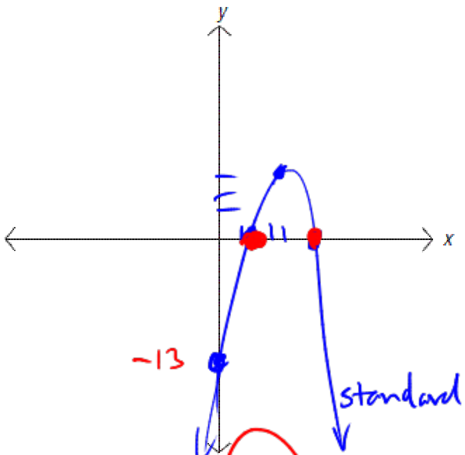
$\left(-b/2a, \; \_\right)$

$= \text{Area}$

$\frac{-2000}{2(-2)} = 500$

what friends?

11. Fill in the blank: I tell my friends this class is ________________



y-int: $x=0$

$-(0-4)^2+3 = -13$

x-int: $y=0$

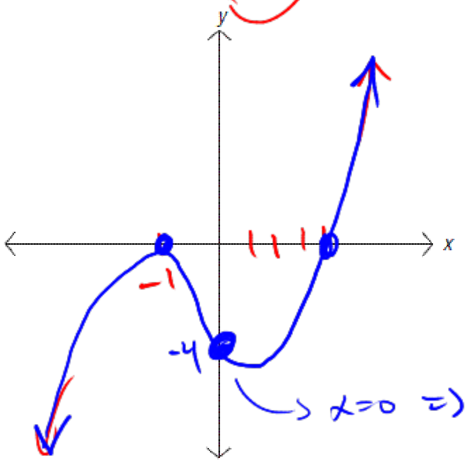
$(x-4)^2 = 3$

$x = 4 \pm \sqrt{3}$

standard form: $(4,3)$. parabola (2)

leading term < 0

(a) $f(x) = -(x-4)^2 + 3$



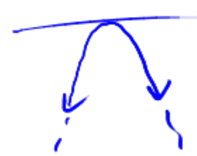
$\to x=0 \Rightarrow y=-4$

Degree 3

leading term >0

zeros: $x=-1$ (multiplicity 2)

$x=4$



(b) $f(x) = (x+1)^2(x-4)$

8. Sketch a graph of the functions above, clearly showing clearly all $x-$ and $y-$intercepts, and the vetex, if appropriate.