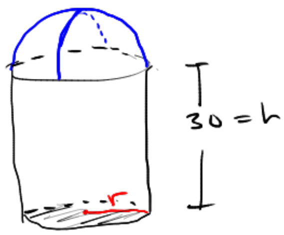
30 , 15 000



Total volume : 15,000 ft^3

Relate the volume to radius

$$V = \underbrace{\text{vol in cylinder}} + \frac{1}{2}\cdot \text{sphere}$$

$$= \pi r^2 30 + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

plot

find

$$15000 = \frac{2\pi}{3} r^3 + 30\pi r^2$$

solve

generic complex #

$$a + bi$$

a is real
b is real

ex.

1

i

$1+i$

$2+i$

$\pi+i$

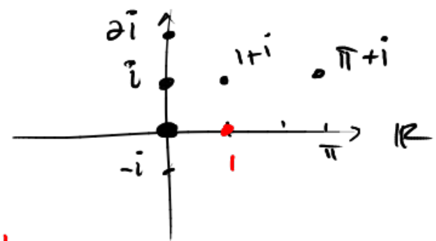
mult. by i $\longleftrightarrow$ rotation

$1 \cdot i = i$

$i \cdot i = -1$

$-1 \cdot i = -i$

$-i \cdot i = -i^2 = -(-1) = 1$



The natural number $e \approx 2.71$. Performs surprisingly well when attempting to model natural phenomena.

variable is in the exponent.

- $f(x) = e^x$. exponential function.

$g(x) = 100 \cdot e^x$

$h(x) = 2^x$ base.

$k(x) = \left(\frac{1}{2}\right)^x = \frac{1^x}{2^x} = \frac{1}{2^x} = 2^{-x}$

$f(x) = e^x$. as $x \to \infty$, $f(x) \to \infty$ very quickly

exponential growth

$d(x) = e^{-x} = \frac{1}{e^x}$ as $x \to \infty$, $d(x) \to 0$

exponential decay

The inverse function of e^x is $\ln(x) = \log_e x$

2^x inverse is $\log_2 x$

10^x inverse is $\log_{10} x$.

Def: $\log_a x = y$ same, same, same as $a^y = x$

logarithmic equation — exponential equation.

$\log_2\left(\frac{1}{8}\right) = ?$ -3

To solve this

$2^x = \frac{1}{8}$ solve this

$2^{-3} = \frac{1}{8}$ so -3 is answer.

$\log_e 4 = x \iff \boxed{e^x = 4}$ $A = x$, $B = 4$

① $\ln 4 = x$ same as $e^A = B$ find A, B.

② what is the exponential form of $\log_{16} 2 = \frac{1}{4} \iff \boxed{16^{1/4} = 2}$

③ Find x. $\log_x 343 = 3. \iff x^3 = 343$

$x = \sqrt[3]{343} = 7.$

we took the cube root

④ $3^x = 343$ can't take the x^{th} root.

solve by applying the inverse function.

The inverse function of 3^x is $\log_3 x$

$\log_3(3^x) = \log_3(343)$

$\log_3(3^x) = x$

$x = \log_3(343)$

$\log_3(3^x) =$ exponent you must raise the base (3) so that it equals 3^x — has to be x.

(or) $\log_3(3^x) = y \iff 3^y = 3^x$ so $x = y$

$f(x) = C a^x$ exponential function.

C is often an initial population.

always means $x=0$ starting point.

passes thru

$(0,3)$ & $(3,24)$

x f(x) x f(x)

find C & a.

① Plug in $(0,3) \rightarrow f(0) = C \cdot a^0 = 3$ so $C=3$.

=1

② now do: $(3,24) \rightarrow f(3) = 3 \cdot a^3 = 24$

solve $\frac{3 \cdot a^3}{3} = \frac{24}{3} \Rightarrow a^3 = 8$

③ # so we cube root

$a = 2$

$$\boxed{f(x) = 3 \cdot 2^x}$$

Questions?

3-D version is

Total Vol = 15,000 ft^3

Bring in the radius into volume formula

$Vol = 15{,}000$

$\underbrace{\text{Vol of cylinder}} + \frac{1}{2}\underbrace{(\text{Vol of sphere})}$

$\boxed{\pi r^2 h \quad + \quad \frac{1}{2}\cdot\frac{4}{3}\pi r^3}$

Volume : $\pi r^2 h$

rewriting ...

$V(r) = \frac{2\pi}{3} r^3 + 30\pi r^2 = 15000$

solve for r ...

plot graph. find r that gives height = 15,000

#14

$y = x^3 - 3x^2 - 4x$

y-int : $x = 0$

$\Rightarrow y = 0$

$\boxed{\text{x-int}}$: $y = 0$

$0 = x^3 - 3x^2 - 4x = x(x^2 - 3x - 4)$

if this true $= x(x-4)(x+1)$

then $x = 0$

or $x = 4$ or $x = -1$

as $x \to \infty$ $x^3 - 3x^2 - 4x$ is behaving like $x^3 \to \infty$

degree 3 $\Rightarrow$

$x \to -\infty$ $x^3 - 3x^2 - 4x$ is behaving $x^3 \to -\infty$

(15) looking for degree 3 poly with zeros: $-3, 2, 6$

$(x - (-3))(x-2)(x-6)$

$= \underbrace{(x+3)(x-2)}_{\text{expand}}(x-6)$

$= (x^2 + x - 6)(x-6)$

$= x^3 + x^2 - 6x - 6x^2 - 6x + 36$

$= x^3 - 5x^2 - 12x + 36$

now mult by 2.

$2x^3 - 10x^2 - 24x + 72$

Same zeros

$\boxed{f(x) = \dfrac{-8x^4 + 56x^3 - 7x^2 + 4x + 8}{1x^4 - 7x^3 - 5x^2 + 7x + 8}} = -8$

Horiz. Asy

$y = -8$

$\cancel{-8x^4} + \cancel{56x^3} - 7x^2 + 4x + 8 = \cancel{-8x^4} + \cancel{56x^3} + 40x^2 - 56x - 64$

$0 = 47x^2 - 60x - 72$

$x = \dfrac{60 \pm \sqrt{60^2 + 4(47)(72)}}{94}$

precise up to 5 decimal places.

$P(x) = x^4 - 1x^3 - 4x^2 - 2x - 12$

Factors of 12: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

know rational zeros are among these

$P(1) = 1 - 1 - 4 - 2 - 12 \neq 0 \Rightarrow (x-1)$ is not a factor

1 is not a root.

$P(-1) = 1 - 1 - 4 + 2 - 12 \neq 0$

$P(2) = 16 - 8 - 4\cdot 4 - 8 - 12$

✓ $P(-2) = 16 + 8 - 16 + 4 - 12 = 0$ so $x = -2$ is a rational zero.

repeat until you find 4 or exhaust the list.

you could use $(x+2)$ is a factor, use long division to simplify.

next $P(3)$

#6 $P(x) = (x-7)^6 - 6$

vertical translation

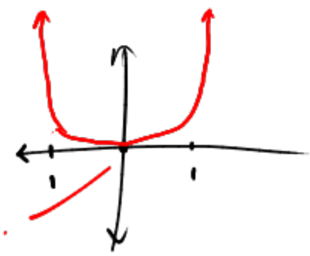
same # of extrema

(doesn't change the # of extrema)

$P_1(x) = (x-7)^6$

horizontal translation

$P_2(x) = x^6$



only 1 local extrema.

just 1 local extrema

$\left\{ \begin{array}{l} x^2 \\ x^4 \\ x^6 \\ x^8 \\ \vdots \\ x^{100} \end{array} \right.$

$f(x) = c\,a^x$, Passes thru $\overset{(x,\ f(x))}{(0,5)}$

& $(3,10)$

Find c & a

$f(0) = c\,\underbrace{a^0}_{=1} = c = 5$

$f(x) = 5 \cdot a^x$

$f(3) = 5 \cdot a^3 = 10$ solve $a^3 = 2$ # take root.

$\sqrt[3]{a^3} = \sqrt[3]{2}$

$a = \sqrt[3]{2}$.

$.

$(i = .05$ for $5\%)$

Invest $\$P$ earning i = interest rate (annual)

You have: $\underbrace{P + P\cdot i}$ $= \underline{P(1+i)}$ ① $\left(\underline{\text{Think: } 100 + \overbrace{100(.05)}^{5} = 105} \right)$ At the end of year 1

$(P + Pi) + \underline{i\,(P+Pi)} = \overbrace{(P+Pi)}^{P(1+i)}(1+i) = P(1+i)^2$ At the end of year 2

what we began with

think 5% of what we began with

exponential function of time. $\boxed{P(1+i)^t}$ year t

Compounded

gets bigger

annually	$P(1+i)^t$
quarterly	$P(1+\frac{r}{4})^{4/r \cdot t}$ ← # years
monthly	$P(1+\frac{r}{12})^{12/r \cdot t}$
daily	$P(1+\frac{r}{365})^{365/r \cdot t}$
continuously	Pe^{rt}

$P(1+\frac{r}{n})^{n/r \cdot t}$

becomes set $m = \boxed{\frac{n}{r}}$

$\boxed{P(1+\frac{1}{m})^{m \cdot t}}$

$(1+\frac{1}{m})^m \longrightarrow e$

as $m \to \infty$

this suggests why we use e^x so often when we're modeling natural behavior (exponential growth (r > 0), exponential decay (r < 0)

nature compounds continuously

$$V(t) = 80\left(1 - e^{-.2t}\right)$$

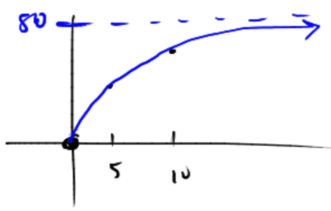
t = seconds
$v(t)$ = ft/sec.

(a) Initial Velocity: $(t=0) \Rightarrow 80(1-e^{0}) = 80(1-1) = \underline{\underline{0}}$. (he fell not dove.)

(b) $V(5) = 80\left(1 - e^{-.2(5)}\right) = 50$ ft/sec $\cdot \frac{3600}{5280} = 34$ m/h

$V(10) = 80\left(1 - e^{-2}\right) = 69$ ft/sec $= 47$ m/h

(c)



80 ft/sec $\times \frac{3600}{5280} = 54$

Horizontal Asymptote: $V(t)$ as $t \to$ BIG

(e) $V(t) = 80\left(1 - \frac{1}{\boxed{e^{.2t}}}\right)$ $\to 0$

WICKED BIG

$= 80$

(d) terminal velocity: 80 ft/sec (horizontal asy.)

(10) I_0 = intensity of a standard earthquake.

I = intensity of some big EQ.

$\boxed{\log_a x = b \text{ same as } a^b = x}$

$$M = \log\left(\frac{I}{I_0}\right)$$

(a) set $I = I_0 \Rightarrow M = \log\left(\frac{I_0}{I_0}\right) = \log_{10}(1) = \boxed{0}$

think $10^x = 1$

(b) An earthquake that's 10 times as intense as I_0 has magnitude:

$$M = \log\left(\frac{10 \cdot \cancel{I_0}}{\cancel{I_0}}\right) = \log_{10} 10 = \boxed{1}$$

$\overset{x=1}{10^x = 10}$

TWO OF SEVERAL PROPERTIES OF LOGS

$\log(A \cdot B) = \log(A) + \log(B)$

$\log\left(\frac{A}{B}\right) = \log(A) - \log(B)$

$$= \log(10) + \log\left(\frac{I_0}{I_0}\right) = \boxed{\log_{10} 10} + \overbrace{\log 1}^{=0}$$

$= 1$

4.1 & 4.2.

(Think: \$/100 $i = .05 = 5\%$)

Money:

Invest \$P into account earning annual interest i.

$P + P \cdot i = \boxed{P(1+i)}$ After year 1

principal & interest earned

$P(1+i) + i \cdot P(1+i) = P(1+i)(1+i) = P(1+i)^2$ After year 2

new principal

$= P(1+i)^3$ After year 3

Total Earnings

$\vdots$ $\vdots$

$\to P(1+i)^n$ After year n

Interest compunded

annually $P(1+i)^n$

quarterly - 4 times/year $P(1+\frac{r}{4})^{\boxed{4} \cdot t}$ $\quad P(1+\frac{r}{4})^{\frac{4}{r} \cdot t}$

exponential function of t, base $1+\frac{r}{4}$

daily - $P(1+\frac{r}{365})^{\frac{365}{n} \cdot t}$

continuously

$$\left(1+\frac{1}{x}\right)^x \longrightarrow e \quad \text{as} \quad x \to \infty.$$

$$P\left(1+\frac{r}{n}\right)^{\frac{n}{r} \cdot t} \longrightarrow Pe^{rt} \quad \text{as} \quad n \to \infty.$$

the function Pe^(rt) is often used to model natural behavior.

this calculation suggests that …. nature compounds continuously

More applications of exponential functions

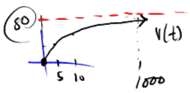
$$V(t) = 80\left(1 - e^{-.2t}\right) = 80\left(1 - \cancel{\frac{1}{e^{.2(t)}}}\right) \to 80\,(1-0) \text{ as } t \to \infty$$

$\to 0$; $= 80$

(a) Initial Velocity : $t = 0$ sub $\longmapsto$ $V(0) = 80\left(1 - e^{0}\right) = 80\,(1-1) = 80 \cdot 0 = 0$

(b) $V(5) = 80\left(1 - e^{-.2 \cdot 5}\right) = 50.5 \frac{ft}{sec} \times \frac{1\ mi}{5280\ ft} \times \frac{3600}{1\ hr} = 34$

$V(10) = 80\left(1 - e^{-2}\right) = 69.1 \frac{ft}{sec} \times \quad = 47$



(c)

(d)

(e)

(3) $\log_a(x) = y$ means $a^y = x$

logarithmic form (equation) — exponential equation

Ex. $\log_2(32) = x$ same as $2^x = 32$ $x = 5$

Ex $\log_a(100) = 2$ same as $a^2 = 100 \overset{\text{square root}}{\Rightarrow} \underline{a = 10}$

6. $\log x$ (no base written, assume base $= 10$)

$\ln x$ base $= e$ $\ln x = \log_e x$

$\log x$ & 10^x are inverse functions.

$\log(10^x) = y$ same as $10^y = 10^x$ so $x = y$.

$= x$

$\log_{10}(10^{x}) = \underline{x}$

cancels

Richter Scale

I_0 = base intensity (background vibration)

$M = \log\left(\frac{I}{I_0}\right) \xrightarrow{\text{set } I = I_0} M = \log\left(\frac{I_0}{I_0}\right) = \log(1) = \underline{0}$

think; $10^x = 1 \Rightarrow x = 0$

Mag of earthquake 10 times as intense as std. quake

$I = 10 I_0 \to M = \log\left(\frac{10 \cancel{I_0}}{\cancel{I_0}}\right)$ $10^x = 10$, $x = 1$

$= \log 10 = 1$

Lastly, logs make big #'s small.

googol $= 10^{100} = 1\,00000000 \underline{\qquad\qquad} 0$

— 100 zeros —

$\log(\log(\text{googol})) = \log(\underbrace{\log(10^{100})}_{\log(100)\, = \,2})$ $\leftarrow 10^{x} = 10^{100}$ $\leftarrow 100$