7.1.5. MA161 – WK1 – Fri

$$\int 3x^3 e^{3x^2}\,dx = \int 3x^3 e^u \frac{1}{6x}\,du = \frac{1}{2}\int x^2 e^u\,du = \frac{1}{2}\int \frac{3x^2}{3} e^u\,du = \frac{1}{2}\cdot\frac{1}{3}\int \underbrace{3x^2}_{u} e^u\,du$$

$\overset{\star}{=} \frac{1}{6}\int u\cdot e^u\,du$

set $\boxed{u = 3x^2}$

$\frac{du}{dx} = 6x$

$du = 6x\,dx$

$\boxed{\frac{1}{6x}du = dx}$

almost $= u$

(good : b/c $\boxed{\int x e^x\,dx}$

ez – I.B.P.

substitution step

abuse notation.

I.B.P.

$\boxed{\text{I.B.P}}$

$u = u \xrightarrow{\text{deriv.}} du = du$

$dv = e^u\,du \xrightarrow{\text{int}} v = e^u$

$$\overset{\star}{=} \frac{1}{6}\int u\cdot e^u\,du = \frac{1}{6}\left[u\cdot e^u - \int e^u\,du\right] = \frac{1}{6}\left[u\cdot e^u - e^u\right]\Big|_{u=3x^2} = \frac{1}{6}\left[3x^2 e^{3x^2} - e^{3x^2}\right]$$

$$= \frac{e^{3x^2}}{6}\left[3x^2 - 1\right] + C$$

This technique: u-sub 1^{st}, then I.B.P.'s solves: $\int e^{\sqrt{x}}\,dx$

$$\int e^{\sqrt{x}}\,dx = \int e^{u}\cdot 2x^{1/2}\,du = \int e^{u}\cdot 2u\,du = 2\int u\cdot e^{u}\,du$$

similar to previous

1^{st}: u-sub

$\boxed{u=\sqrt{x}}$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2}$

$du = \frac{1}{2}x^{-1/2}\,dx$

$\boxed{2x^{1/2}\,du = dx}$

$u = u \qquad dv = e^{u}$

$du = du \qquad v = e^{u}$

$= 2\left[ue^{u} - \int e^{u}\,du\right]$

$= 2\left[\sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}}\right] + c$

Similar to this:

$\int \cos(\sqrt{x})\,dx$

$\int \sin(\sqrt{x})\,dx$

$\int \ln(\sqrt{x})$

Ex.

$$\int \ln(\sqrt{x})\,dx = \int \ln(u)\, 2\sqrt{x}\, du = 2\int \ln(u)\cdot u\, du \overset{\text{step 2}}{=} 2\left[\ln(u)\cdot\frac{u^2}{2} - \frac{u^2}{4}\right] = 2\left[\frac{\ln(\sqrt{x})\,x}{2} - \frac{x^4}{4}\right] + C$$

$u = \sqrt{x}$ $\quad u^2 = x$!

step 1 u-sub 1st

$du = \frac{1}{2}x^{-1/2}\,dx$

$2\sqrt{x}\,du = dx$

step 2:

$u = u \xrightarrow{d/dx} du = du$

$dv = \ln(u)\,du \xrightarrow{\int} \int dv = v = \underbrace{\int \ln(u)\,du}_{\text{I.B.P.}}$

LIPET

$u = \ln(u) \quad dv = du$

$du = \frac{1}{u}\,du \quad v = u$

$= u\cdot\ln(u) - \int \underbrace{u\cdot\frac{1}{u}\,du}_{u}$

$= \boxed{u(\ln(u) - 1)}$

$$2\left[u\left[u(\ln(u)-1)\right] - \int u(\ln(u)-1)\,du\right]$$

$$2\left[u\left[u(\ln(u)-1)\right] - \int u\ln(u) - u\,du\right]$$

another I.B.P.

better step 2

$u = \ln(u) \quad dv = u\,du$

$du = \frac{1}{u}\,du \quad v = \frac{u^2}{2}$

$= \ln(u)\cdot\frac{u^2}{2} - \int \frac{u^2}{2}\cdot\frac{1}{u}\,du$

$= \ln(u)\cdot\frac{u^2}{2} - \frac{1}{2}\int u\,du$

$= \ln(u)\cdot\frac{u^2}{2} - \frac{u^2}{4}$

TRIG I.B.P. Problems

Ex $\int x \sin x \, dx = -x\cos x + \int \cos x \, dx = \underset{\text{product}}{-x\cos x} + \sin x + C$

$u = x \qquad dv = \sin x \, dx$ $\qquad \frac{d}{dx}(\text{ans}) = -\cos x + x\sin x + \cos x = x \cdot \sin x$:)

$du = dx \qquad v = \int \sin x \, dx = -\cos x$

Ex $\int x^2 \sin x \, dx = -x^2\cos x + 2\int x\cos x \, dx = -x^2\cos x + 2\left[x\sin x - \overbrace{\int \sin x \, dx}^{-\cos x}\right]$

$u = x^2 \qquad dv = \sin x$ | $u = x \qquad dv = \cos x$

$du = 2x \, dx \qquad v = -\cos x$ | $du = dx \qquad v = \sin x$

$= -x^2\cos x + 2x\sin x + \cos x + C$

$= \boxed{(1 - x^2)\cos x + 2x \cdot \sin x + C}$