MA163 – wk 1 TUE

Next 3 weeks: Integration

Today: u-sub, Integration By Parts

Recall: $\int f \pm g = \int f \pm \int g$

$\int f \cdot g \neq \int f \cdot \int g$

ex u-sub: when in doubt, set u = what is inside radical, or inside parentheses

$$\int x\sqrt{3x+8}\,dx = \int x\sqrt{u}\cdot\frac{1}{3}\,du = \int \frac{1}{3}(u-8)\sqrt{u}\,\frac{1}{3}\,du = \frac{1}{9}\int (u-8)\sqrt{u}\,du$$

(algebra)

integrand is mixed … bad

1. $u = 3x+8$

2. $\frac{d}{dx}(u) = \frac{d}{dx}(3x+8)$

$\frac{du}{dx} = 3$

treat LHS as a quotient of differentials

3. $du = 3dx$

4. $\frac{1}{3}du = dx$

5. get x in terms of u

isolate x in 1

$u - 8 = 3x$

$\frac{1}{3}(u-8) = x$

$$= \frac{1}{9}\int u^{3/2} - 8u^{1/2}\,du \quad \text{power rule}$$

$$= \frac{1}{9}\left[\frac{u^{5/2}}{\frac{5}{2}} - \frac{8u^{3/2}}{\frac{3}{2}}\right] + C$$

– get x back –

$$= \frac{2}{45}\cdot u^{5/2} - \frac{16}{27}u^{3/2} + C$$

$$= \frac{2}{45}(3x+8)^{5/2} - \frac{16}{27}(3x+8)^{3/2} + C$$

ex $\int 5(3x+7)^2\,dx$

$\int u^2\,du$

$u = 3x+7$ | $du = 3dx$

$\frac{du}{dx} = 3$ | $\frac{1}{3}du = dx$

$$= 5\int u^2\,\frac{1}{3}\,du = \frac{5}{3}\int u^2\,du = \frac{5}{3}\frac{u^3}{3} + C = \frac{5(3x+7)^3}{9} + C$$

Basic Integrals Chart

Functions	Anti-Derivative	Functions	Anti-Derivative
u^n	$\frac{u^{n+1}}{n+1}$	$\sec(u)\tan(u)$	$\sec(u)$
		$\csc(u)\cot(u)$	$-\csc(u)$
$\frac{1}{u}$	$\ln\lvert u\rvert$	$\frac{1}{1+u^2}$	$\tan^{-1}(u)$
e^u	e^u		
		$\frac{1}{\sqrt{1-u^2}}$	$\sin^{-1}(u)$
$\sin(u)$	$-\cos(u)$		
$\cos(u)$	$\sin(u)$		
$\sec^2(u)$	$\tan(u)$	$\frac{1}{\lvert u\rvert\sqrt{u^2-1}}$	$\sec^{-1}(u)$
$\csc^2(u)$	$-\cot(u)$		

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why $\boxed{\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}}$

Set * $y = \tan^{-1}x$

Find $\frac{dy}{dx}$

① Hit* w/ inverse fcn'

$\tan(y) = x$

② Apply $\frac{d}{dx}$, remember chain rule

$\sec^2(y)\cdot\frac{dy}{dx} = 1$

③ $\frac{dy}{dx} = \frac{1}{\sec^2(y)}$

④ use Pythag Trig Id to get x back $\quad \sec^2(y) = \frac{1}{\cos^2(y)}$

$\sin^2(y) + \cos^2(y) = 1$

$\div\cos^2(y) \Rightarrow \tan^2(y) + 1 = \sec^2(y)$

⑤ putting ④, ③ & ① together:

$\frac{dy}{dx} = \frac{1}{x^2+1} = \frac{1}{1+x^2}$

Preview of Int. By Parts

$$(u \cdot v)' = u' \cdot v + u \cdot v'$$

isolate this

$$u \cdot v' = (u \cdot v)' - u' \cdot v$$

integrate both sides

$$\int u \cdot v' = \int (u \cdot v)' - u' \cdot v = \int (u \cdot v)' - \int u' \cdot v$$

$$\boxed{\int u \cdot v' = uv - \int v \cdot u'}$$ Int. By Parts

Ultra Violet minus Super Voo Doo`