Integration By Parts   wk 1 – Th

Last Time.

$v' =$ differential $= dv$

small (infinitesimal) change in v

$$\boxed{\int u \cdot v' = u \cdot v - \int v \cdot u'}$$

$$\|$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

ultra - violet minus super voo doo

Ex.

$$\int x e^x dx$$

Everything behind the $\int$ gets used

$$\boxed{u = x} \qquad \boxed{dv = e^x dx}$$

$\downarrow$ apply $\frac{d}{dx}$ $\qquad$ $\downarrow$ apply $\int$

$$\boxed{du = dx} \qquad \boxed{v = e^x}$$

notice the dx vanishes @ this step

$$\int 1\,dx = x$$

$$\int dv = v$$

$$\underbrace{\int x e^x dx}_{\text{question}} \overset{\text{set}}{=} \int u\,dv = \overset{\text{Formula}}{uv - \int v\,du} := xe^x - \int e^x dx = xe^x - e^x + C = \underbrace{\boxed{e^x(x-1) + C}}_{\text{ans.}}$$

Ex.

$$\int (4x+3)e^x \underline{\underline{dx}} = e^x[2x^2 + 3x] - \int \left(\frac{4x^2}{2} + 3x\right) e^x dx$$

see product
think I.B.P.

try: $u = e^x \qquad dv = (4x+3)dx$

$\downarrow \qquad\qquad \downarrow$

$du = e^x dx \qquad v = \int (4x+3)dx = \frac{4x^2}{2} + 3x$

Important Point:

I. B. P. is an algorithm that breaks one complicated problem into more simpler ones.

Here, we see: our solution requires a more complicated integral than we had at the beginning.

Try again.

no need to add C on these intermediate steps, one anti-derivative will suffice

often:
set u = something whose derivative simpler than it is.

again: $u = 4x+3 \qquad dv = e^x dx$

$du = 4\,dx \qquad v = e^x$

$$\text{ans} = (4x+3)e^x - \int e^x \cdot 4\,dx = (4x+3)e^x - 4e^x + C = e^x[4x-1] + C$$

$$\text{check: } \frac{d}{dx}(\text{ans}) = e^x[4x-1] + e^x \cdot 4 = 4xe^x - e^x + 4e^x = 4xe^x + 3e^x = e^x[4x+3]$$

HINT FOR CHOOSING u	

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LIPET , log, inverse trig, power, exponential, trig

LIATE , log, inverse trig, algebraic, trig, exp

Point: Integration of functions is hard. Not every integral can be solved by hand.

Ex

(A) $\int 5x^3 e^{5x^2}\,dx = 5x^3$

$u = 5x^3$ $\qquad dv = e^{5x^2}\,dx$

$du = 15x^2\,dx \qquad v = \int e^{5x^2}\,dx$ ← this one has no closed form sol'n.

(only way to integrate is via numerical approx

Ex

$\int 5e^{\sqrt{x}}\,dx = 5xe^{\sqrt{x}} - \int \frac{5}{2}e^{\sqrt{x}}\sqrt{x}\,dx$

Is this easier than before?
(not u-sub)

$u = 5e^{\sqrt{x}} \qquad dv = dx$

$du = 5e^{\sqrt{x}}\cdot\frac{1}{2}x^{-1/2} \qquad v = x$

this is similar to previous

Ex

LIPET

$$\int \ln(x)\,dx = x\ln x - \int \underbrace{x\cdot\frac{1}{x}}_{=1}\,dx$$

$u = \ln(x)$ $\quad dv = dx$

$du = \frac{1}{x}dx$ $\quad v = x$

$$= x\ln x - \int 1\,dx = x\ln x - x + C$$

$$\frac{d}{dx}(x\ln x - x) = 1\cdot\ln x + x\cdot\frac{1}{x} - 1 = \ln x$$

Ex $\displaystyle\int x^2 e^x\,dx = x^2e^x - \underbrace{\int e^x 2x\,dx}_{\text{IBP}} = x^2e^x - \left[2xe^x - \underbrace{\int e^x 2\,dx}_{2e^x}\right]$

$u = x^2$ $\quad dv = e^x dx$

$du = 2x\,dx$ $\quad v = e^x$

OK to I.B.P. mult. times on same problem

$u = 2x$ $\quad dv = e^x dx$

$du = 2dx$ $\quad v = e^x$

Point:

sometimes multiple I.B.P.'s are necessary`

$$= x^2e^x - 2xe^x + 2e^x + C$$

$$= \boxed{e^x(x^2 - 2x + 2) + C}$$