

MA163 wk-1 Wed

HW (u-sub)

$$\int x \sqrt{3x+8} dx \longrightarrow \int u du$$

key: power inside $\sqrt{\quad}$ = 1 = same as degree outside
(degree)

$$\sqrt{\quad} = (\quad)^{1/2}$$

$$\text{step 1} = \int x(3x+8)^{1/2} dx \xrightarrow{\text{step 5}} \int x(u)^{1/2} \frac{1}{3} du \xrightarrow{\text{step 6}} \int \left[\frac{u-8}{3}\right] u^{1/2} \frac{1}{3} du = \frac{1}{9} \int (u-8) u^{1/2} du$$

mixed variables

$$\text{step 2} \quad \boxed{u = 3x+8} \rightarrow \frac{u-8}{3} = x$$

$$\text{step 3} \quad \frac{d}{dx}(u) = \frac{d}{dx}(3x+8)$$

$$\frac{du}{dx} = 3$$

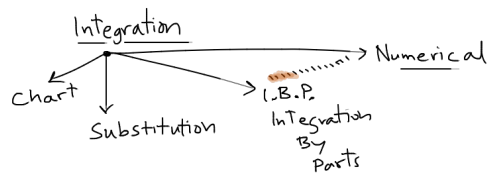
$$\text{step 8} = \frac{1}{9} \int u^{3/2} - 8u^{1/2} du$$

power rule

$$\text{step 9} = \frac{1}{9} \left[\frac{2}{5} u^{5/2} - \frac{2}{3} 8 \cdot u^{3/2} + C \right] \xrightarrow{\text{step 10}} \boxed{\frac{2}{45} (3x+8)^{5/2} - \frac{16}{27} (3x+8)^{3/2} + C}$$

get back

$$\text{step 4} \quad du = 3dx \Rightarrow \boxed{\frac{1}{3} du = dx} \quad (\text{isolate } dx)$$



Basic Integrals Chart

Functions	$\int \frac{1}{u} du$ Anti-Derivative	$u = f(x)$	Functions	Anti-Derivative
u^n	$\frac{u^{n+1}}{n+1}$		$\frac{1}{1+u^2}$	$\tan^{-1}(u)$ ★
$\frac{1}{u}$	$\ln u $		$\frac{1}{\sqrt{1-u^2}}$	$\sin^{-1}(u)$
e^u	e^u		$\frac{1}{ u \sqrt{u^2-1}}$	$\sec^{-1}(u)$
$\sin(u)$	$-\cos(u)$			
$\cos(u)$	$\sin(u)$			
$\sec^2(u)$	$\tan(u)$			
$\sec(u)\tan(u)$	$\sec(u)$			
$\csc^2(u)$	$-\cot(u)$			
$\csc(u)\cot(u)$	$-\csc(u)$			

$$\frac{d}{dx} \cot(u) = \frac{d}{dx} \frac{1}{\tan(u)} = \frac{\tan(x) \cdot 0 - 1 \cdot \sec^2 x}{\tan^2 x} = \frac{-\sec^2 x}{\tan^2 x}$$

$$= \frac{-\frac{1}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x}} = -\frac{1}{\sin^2 x} = -\csc^2 x$$

$$\Rightarrow \int \csc^2 x dx = -\cot(x)$$

★ Why $\frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2}$?

① set: $y = \tan^{-1} x$, want $\frac{dy}{dx} =$ _____

② hit w/ inverse function

$$\tan(y) = \tan(\tan^{-1} x) = x$$

③ apply $\frac{d}{dx}$ to both sides:

$$\frac{d}{dx} (\tan(y)) = \frac{d}{dx} (x) = \frac{dx}{dx} = 1$$

$$\downarrow$$

$$\sec^2(y) \cdot \frac{dy}{dx}$$

$$\sec^2(y) \cdot \frac{dy}{dx} = 1$$

Trig
Pyth Thm:

$$\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$$

$$\downarrow$$

$$\tan^2 + 1 = \sec^2$$

want x back

$$\frac{dy}{dx} = \frac{1}{\sec^2(y)}$$

$$= \frac{1}{\tan^2 y + 1}$$

$$\star = \frac{1}{x^2 + 1}$$

Integration By Parts

Recall the product rule for derivatives

$$(u \cdot v)' = u' \cdot v + \underbrace{u \cdot v'}$$

use $()' = \frac{d}{dx}$

isolate this $\Rightarrow$

$$u \cdot v' = (u \cdot v)' - u' \cdot v$$

) integrate both sides

$$\int u \cdot v' = \int [(u \cdot v)' - u' \cdot v]$$

$$\downarrow = \int \underbrace{u \cdot v'} - \int \underbrace{u' \cdot v}$$

$\int u \cdot v' = u \cdot v - \int v \cdot u'$

I.B.P. Formula

easier
than
D.G

Ex

$$\int x \cdot e^x dx$$

goal

match $\rightarrow$