MA163 wk-1 Wed

HW (u-sub)

$\int x\sqrt{3x+8}\,dx \longrightarrow \int u^n\,du$

key: power inside $\sqrt{}$ (degree) $= 1 =$ same as degree outside

$\sqrt{} = (\ \ )^{1/2}$

step 1 $= \int x(3x+8)^{\frac{1}{2}}\,dx$ $=\!=$ step 5 $\underbrace{\int x(u)^{\frac{1}{2}}\frac{1}{3}du}_{\text{mixed variables}}$ $=$ step 6 $\int \left[\frac{u-8}{3}\right] \cdot u^{\frac{1}{2}}\frac{1}{3}du = \frac{1}{3}\int[u-8]\cdot u^{\frac{1}{2}}\frac{1}{3}du$ ⑦

step 2 $\boxed{u = 3x+8} \longrightarrow \frac{u-8}{3} = x$

⑧ $= \frac{1}{9}\int u^{\frac{3}{2}} - 8u^{\frac{1}{2}}\,du$

power rule

step 3 $\frac{d}{dx}(u) = \frac{d}{dx}(3x+8)$

$\frac{du}{dx} = 3$

⑨ $= \frac{1}{9}\left[\frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}8\cdot u^{\frac{3}{2}} + C\right] =$ ⑩ $\boxed{\frac{2}{45}(3x+8)^{\frac{5}{2}} - \frac{16}{27}(3x+8)^{\frac{3}{2}} + C}$

get x back

step 4 $du = 3dx \Rightarrow \boxed{\frac{1}{3}du = dx}$ (isolate dx)

Integration
- Chart
- Substitution
- I.B.P. Integration By Parts
- Numerical

Basic Integrals Chart

Functions	Anti-Derivative $\int \frac{1}{u}du$	Functions $u=f(x)$	Anti-Derivative
u^n	$\frac{u^{n+1}}{n+1}$	$\frac{1}{1+u^2}$	$\tan^{-1}(u)$ ⊛
$\frac{1}{u}$	$\ln\lvert u\rvert$	$\frac{1}{\sqrt{1-u^2}}$ 2nd	$\sin^{-1}(u)$
e^u	e^u	$\frac{1}{\lvert u\rvert\sqrt{u^2-1}}$ 1st	$\sec^{-1}(u)$
$\sin(u)$	$-\cos(u)$		
$\cos(u)$	$\sin(u)$		
$\sec^2(u)$	$\tan(u)$		
$\sec(u)\tan(u)$	$\sec(u)$		
$\csc^2(u)$	$-\cot(u)$		
$\csc(u)\cot(u)$	$-\csc(u)$		

$$\frac{d}{dx}\cot(u) = \frac{d}{dx}\frac{1}{\tan(u)} = \frac{\tan(x)\cdot 0 - 1\cdot\sec^2 x}{\tan^2 x} = \frac{-\sec^2 x}{\tan^2 x}$$

$$= \frac{-\frac{1}{\cancel{\cos^2 x}}}{\frac{\sin^2 x}{\cancel{\cos^2 x}}} = -\frac{1}{\sin^2 x} = -\csc^2 x$$

$$\Rightarrow \int \csc^2 x\,dx = -\cot(x)$$

⊛ Why $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$?

① set: $y=\tan^{-1}x$, want $\frac{dy}{dx} =$ ______

② hit w/ inverse function: ⊛ $\boxed{\tan(y) = \tan(\tan^{-1}x) = x}$

Trig Pyth Thm:

$$\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$$

$$\tan^2 + 1 = \sec^2$$

③ apply $\frac{d}{dx}$ to both sides: $\frac{d}{dx}(\tan(y)) = \frac{d}{dx}(x) = \frac{dx}{dx} = 1$

$$\downarrow$$

$$\sec^2(y)\cdot\underbrace{\frac{d}{dx}(y)}_{\frac{dy}{dx}}$$

$$\sec^2(y)\cdot\frac{dy}{dx} = 1$$

want x back

$$\frac{dy}{dx} = \frac{1}{\sec^2(y)}$$

$$= \frac{1}{\tan^2 y + 1}$$

$$⊛ = \frac{1}{x^2+1}$$

Integration By Parts

Recall the product rule for derivatives

$$(u \cdot v)' = u' \cdot v + \underbrace{u \cdot v'}$$

isolate this

$$\boxed{\text{use } (\ )' = \frac{d}{dx}}$$

$$u \cdot v' = (u \cdot v)' - u' \cdot v$$

integrate both sides

$$\int u \cdot v' = \int \left[(u \cdot v)' - u' \cdot v \right]$$

$$= \int (u \cdot v)' - \int \underbrace{u' \cdot v}$$

$$\boxed{\int u \cdot v' = u \cdot v - \int v \cdot u'}$$

I.B.P. Formula

easier than O.G

Ex

goal

match

$$\int x \cdot e^x dx$$