wk 10 - thur

Find power series representation: use Maclaurin series (it is a power series)

$f(x) = \dfrac{8x}{(x^2+1)^2}$

Hint: start w/ $g(x) = (1+x^2)^{-1} = \dfrac{1}{1+x^2}$, then differentiate

$= \dfrac{1}{1-\underbrace{(-x^2)}_{r}} \overset{\text{geometric}}{=} \dfrac{1}{1-r}$

$= 1 + r + r^2 + r^3 + \ldots$

1. True

$$\boxed{\dfrac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \ldots}$$

$\frac{d}{dx}$

d/dx(LHS)

$-(1+x^2)^{-2} \cdot 2x = -2x + 4x^3 - 6x^5 + 8x^7 - \ldots$

$$\frac{-2x}{(1+x^2)^2} = \sum_{n=0}^{\infty} (-1)^{n+1} 2(n+1)\, x^{2n+1}$$

Assume True

get f(x) desired by mult by -4

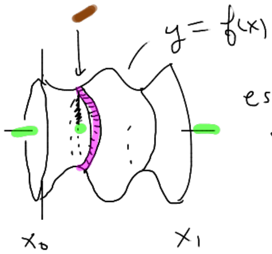
$$(-4)\cdot\frac{-2x}{(1+x^2)^2} = \frac{8x}{(1+x^2)^2} = -4\sum_{n=0}^{\infty}(-1)^{n+1}2(n+1)\,x^{2n+1} = \sum_{n=0}^{\infty}(-1)^{n+1}\underbrace{(-8)(n+1)}_{8(1-n)}\,x^{2n+1}$$

set $m = n-1$

$$= \sum_{m=0}^{\infty}(-1)^{m+1}(-8)(m+1)\,x^{2m+1} = \sum_{n=1}^{\infty}\overbrace{(-1)^{n}}^{(-1)^{n+1}}(-1)(8)(n)\,x^{2n-1}$$

$2(n-1)+1 = 2n-1$

Surface Area of Surfaces of Revolution.



estimate area of band and then add all bands

take limit

width = arclength

radius = f(x)

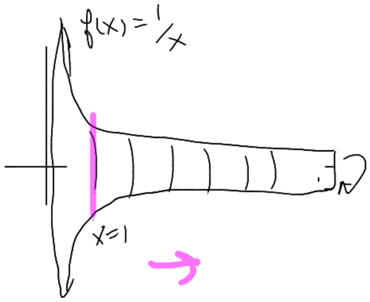
Area of band = circumference × width

$= 2\pi r \cdot \text{width}$

$= 2\pi f(x) \cdot \underbrace{\text{arclength}}_{\sqrt{1+(f'(x))^2}}$

$2\pi f(x)\sqrt{1+(f'(x))^2}$

$$A \int_{x_0}^{x_1} 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx$$



what is the area from x=1 all the way to right (∞)

$\frac{1}{x} = x^{-1}$

$-x^{-2}$

$$\int_1^\infty 2\pi\left(\frac{1}{x}\right)\sqrt{1+\left(\frac{-1}{x^2}\right)^2}$$

$\overset{\shortparallel}{\frac{x^4}{x^4}} + \overset{\shortparallel}{\frac{1}{x^4}}$

$=$

$$= \int_0^1 2\pi\,\frac{1}{x}\sqrt{\frac{x^4+1}{x^4}}\,dx$$

$$= \int_0^1 2\pi\,\frac{\sqrt{x^4+1}}{x^3}\,dx > \int_0^1 2\pi\,\frac{\sqrt{x^4}}{x^3}\,dx$$

$$= 2\pi\int_0^1 \frac{1}{x}\,dx = 2\pi \ln|x|\Big|_0^1$$

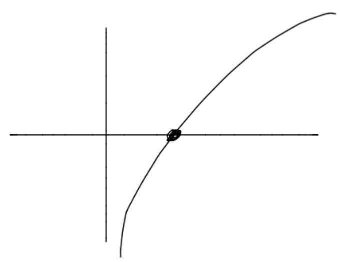
$$= 2\pi\,|\ln(1) - \ln(0)|$$

$$= 2\pi\left[(0) - (-\infty)\right]$$

$$= \infty$$

$\cancel{e^{\ln(1)}} = e^0 = 1$

$\ln(1) = 0$

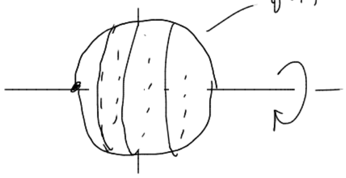


Area of Sphere

$x^2 + y^2 = 1$

4π

$f(x) = \sqrt{1 - x^2}$



$$A = \int_{-1}^{1} 2\pi \left(\sqrt{1-x^2}\right) \sqrt{1 + \left(\frac{x}{\sqrt{1-x^2}}\right)^2}$$

$$f' = \frac{1}{2}(1 - x^2)^{-1/2}(-2x)$$

$$= \frac{x}{\sqrt{1-x^2}}$$

$$A = \int_{-1}^{1} 2\pi \sqrt{1-x^2} \sqrt{\frac{1-\cancel{x^2}}{1-x^2} + \frac{\cancel{x^2}}{1-x^2}}$$

$$A = \int_{-1}^{1} 2\pi \cancel{\sqrt{1-x^2}} \frac{1}{\cancel{\sqrt{1-x^2}}} dx = A = \int_{-1}^{1} 2\pi \, dx$$

$$= 2\pi x \Big|_{-1}^{1}$$

$$= 2\pi(1) - (2\pi(-1)) = 2\pi + 2\pi = 4\pi$$