Wk 10 – Wed.

Exam 3 = Next Wed.

Topics: Taylor + MacLaurin Series, (Estimation)
Arc Length + Surface Area

ex

$$\int_0^1 2\tan^{-1}(x^2)\,dx = (\star)$$

Plan: ① series for $\tan^{-1}(x)$

② sub in x^2 into series

③ sub ①+② into integral

④ use 'linearity' (properties of how $\int dx$ interacts w/ + & scalar mult.

MacLaurin Series

$$\frac{1}{1-r} = 1+r+r^2+r^3$$

$$\frac{1}{1-(-x)} = 1-x+x^2-x^3 \dots$$

$$\frac{1}{1-(-x^2)} = 1-x^2+x^4-x^6 \dots$$

① $\tan^{-1}x = \int \frac{1}{1+x^2}dx = \int \underbrace{\frac{1}{1-(-x^2)}}_{\text{geometric series}} dx$

$$= \int 1-x^2+x^4-x^6+\dots\,dx$$

$$= x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$$

when series, converge - manipulations are valid

② $\tan^{-1}(x^2) = x^2 - \frac{x^6}{3} + \frac{x^{10}}{5} - \frac{x^{14}}{7} + \dots$

$$\int f(x) + g(x) = \int f(x) + \int g(x)$$

③ $(\star) = 2\int_0^1 \tan^{-1}(x^2) = 2\int_0^1 x^2 - \frac{x^6}{3} + \frac{x^{10}}{5} - \frac{x^{14}}{7} + \dots\,dx = 2\int_0^1 \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{2n+1}\,dx$

④ $\underbrace{2\sum_{n=0}^{\infty}(-1)^n \int_0^1 \frac{x^{4n+2}}{2n+1}\,dx}$ $= 2\sum_{n=0}^{\infty}(-1)^n \frac{x^{4n+3}}{(4n+3)(2n+1)}\Big|_{x=0}^{x=1}$

$$\int c f(x) = c\int f(x)$$

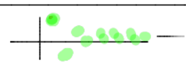
b/c $\int x^2 - \int \frac{x^6}{3} + \int \frac{x^{10}}{5} - \int \frac{x^{14}}{7}$

$\frac{x^3}{3} - \frac{x^7}{21} + \frac{x^{11}}{55} - \frac{x^{15}}{105} + \dots$

$$= 2\sum_{n=0}^{\infty}(-1)^n \left[\frac{1}{(4n+3)(2n+1)} - \frac{0}{(4n+3)(2n+1)}\right]$$

$$= \sum_{n=0}^{\infty}(-1)^n \frac{2}{(4n+3)(2n+1)}$$

(part II)



this is an alternating series, the error incurred by approximating using only n terms is always by the (n+1) term:

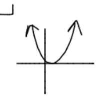
sub (n+1) ms soln term: $\frac{2}{(4(n+1)+3)(2(n+1)+1)} = \frac{2}{(4n+7)(2n+3)}$ set $< \frac{10^{-5}}{1}$

Error is in n^{th} approx less than this

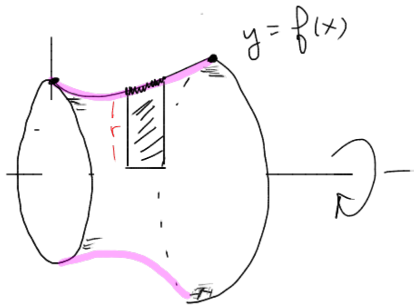
solve this inequality:

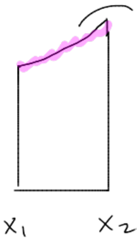
① cross mult.

$2 < 10^{-5}\underbrace{(4n+7)(2n+3)}_{\text{quadratic deg 2!}} \iff \underset{\text{graph}}{2\cdot 10^5} < \underbrace{8n^2+26n+21}_{\text{graph}}$



Surfaces of Revolution





arclength $L = \int_{x_1}^{x_2} \sqrt{1 + [f'(x)]^2}\, dx$

$2\pi r$ = circumference

Surface Area = $\int 2\pi r \underbrace{\sqrt{1 + (f'(x))^2}}_{\text{arclength}}\, dx$