Math 163 - Calculus - Exam 3 - Guide **Name:** ______________________
October 31, 2025
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1. Find the third degree Taylor polynomial for $x^{3/2}$ about x ~~= 1.~~

$(x^{3/2})$

$3x-3$

$\sin(3x-3)$ about $x=1$.

Mac

You may assume: $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!}$

$\sin(x^2)$, similarly

$\sin(x-1) = (x-1) - \frac{(x-1)^3}{3!} + \frac{(x-1)^5}{5} - \ldots$

$\sin(3x-3) = (3x-3) - \frac{(3x-3)^3}{3!} + \frac{(3x-3)^5}{5!} - \ldots$

Mac centered @ $x=0$

Tayl centered @ $x=1$

$\sin(3x-3) = (3x-4) - \frac{(3x-4)^3}{3!} + \frac{(3x-4)^5}{5!}$

2. Find the arc length of the curve

$$y = \frac{2}{3}x^{3/2} \text{ from 0 to 3}$$

Find Taylor Series for $f(x) = x^{3/2}$ @ $\underline{x=1}$

(No manipulations will help – do it from scratch)

$f(x) = x^{3/2}$	$f(1) = 1^{3/2}$	$\frac{f^{n}(1)}{n!}$
$f'(x) = \frac{3}{2}x^{1/2}$	$f'(1) = 3/2$	
$f''(x) = \frac{1}{2}\cdot\frac{3}{2}x^{-1/2}$	$f''(1) = \frac{1}{2}\cdot\frac{3}{2}$	$\div 2!$
$f'''(x) = \frac{1}{2}\frac{3}{2}\frac{-1}{2}x^{-3/2}$	$f'''(1) = \frac{1\cdot 3(-1)}{2^3}$	$\div 3!$
$f^{(iv)} = \frac{1\cdot 3(-1)\cdot(-3)}{2^4}x^{-5/2}$	$f^{iv}(1) = \frac{1\cdot 3(-1)(-3)}{2^4}$	$\div 4!$

$$x^{3/2} = \underset{n=0}{1} + \underset{n=1}{\frac{\frac{3}{2}x^1}{1!}} + \underset{n=2}{\frac{3x^2}{2^2\cdot 2!}} - \underset{n=3}{\frac{3x^3}{2^3\cdot 3!}} + \ldots$$

$$= \sum_{n=0}^{\infty} \frac{3(2n+1)x^n}{2^n\cdot n!}$$

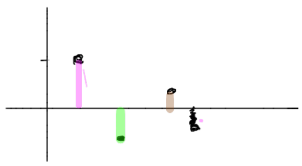
3. The Maclaurin series for $\cos x$ is below.

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{n=0}^{+\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

Find the interval of convergence.

$+ \frac{x^8}{8!}$

ALTERNATING



For Alt. series, the error incurred @ the n^{th} approximation (dist. b/w horiz. line below & n^{th} dot) is less than the next term

4. Use a series to approximate $\cos\left(\frac{1}{2}\right)$ to within 0.01 accuracy.

① $\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!}$

Since the T. series for cos is alt. we can do this!

② $\cos(0.5) = \overbrace{1 - \frac{(0.5)^2}{2!} + \frac{(0.5)^4}{4!}}\; - \frac{(0.5)^6}{6!}$

Since $\frac{(0.5)^6}{6!} \approx 0.00002 < 0.01$ we know!

$1 - \frac{(0.5)^2}{2!} + \frac{(0.5)^4}{4!}$ is within 0.01 of $\cos\left(\frac{1}{2}\right)$

NTK: $\sin x$ $\cos x$ $\ln(x)$

Taylor @ $c=0$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!}$$

$$\cos(x^2) = 1 - \frac{x^4}{2!} + \frac{x^8}{4!} - \frac{x^{12}}{6!} + \frac{x^{16}}{8!}$$

5. Use an eighth degree Taylor polynomial to estimate

$$\int_0^1 \frac{\cos(x^2) - 1}{x}\, dx =$$

$$\frac{\cos(x^2)-1}{x} \approx \frac{-\frac{x^4}{2!} + \frac{x^8}{4!} - \frac{x^{12}}{6!} + \frac{x^{16}}{8!}}{x} = -\frac{x^3}{2!} + \frac{x^7}{4!} - \frac{x^{11}}{6!} + \frac{x^{15}}{8!}$$

$$\approx \int_0^1 -\frac{x^3}{2!} + \frac{x^7}{4!} - \frac{x^{11}}{6!} + \frac{x^{15}}{8!}\, dx = \frac{-x^4}{4\cdot 2!} + \frac{x^8}{8\cdot 4!} - \frac{x^{12}}{12\cdot 6!} + \frac{x^{16}}{16\cdot 8!}\Bigg|_0^1 = \frac{-1}{4\cdot 2!} + \frac{1}{8\cdot 4!} - \frac{1}{12\cdot 6!} + \frac{1}{16\cdot 8!}$$