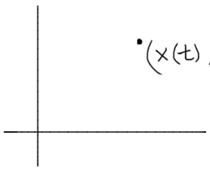
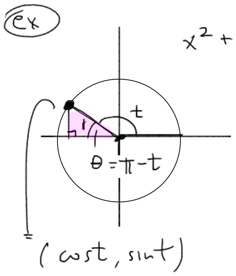
thur. wk 11

Parametric Equations : depend on an input (parameter



ex



$x^2 + y^2 = 1$ ⇝ unit circle

$x^2 + y^2 = R^2$ ⇝ circle, w/ radius R

unit circle parametric eq's	radius R
$x(t) = \cos t$	$x(t) = R\cos t$
$y(t) = \sin t$	$y(t) = R\sin t$

ex parametric eq's

$x(t) = 5\cos(t)$

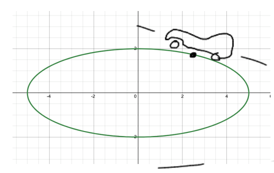
$y(t) = 2\sin(t)$

we can find $\frac{dx}{dt}$, $\frac{dy}{dt}$

$\frac{dx}{dt} = x'(t) = -5\sin t$ speed of the x-coord

$\frac{dy}{dt} = 2\cos t$ speed of the y-coord

we also will want $\frac{dy}{dx}$ {slope of tangent line}



method #1

$$\frac{dy}{dx} = \underbrace{\frac{dy}{dt}\,\frac{dt}{dx}}_{\text{chain rule}} \stackrel{\text{view as fraction}}{=} \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2\cos t}{-5\sin t} = -\frac{2}{5}\cot(t)$$

method #2 "eliminate the parameter"

(need the $x(t)$, $y(t)$ functions to be invertible)

ex $x(t) = 5t - 1$

$y(t) = \frac{1}{t}$

" eliminate the parameter "

solve for t in $x(t)$

① set $x = x(t)$

$x = 5t - 1$

② solve for t

$x + 1 = 5t$

③ $\frac{x+1}{5} = t$

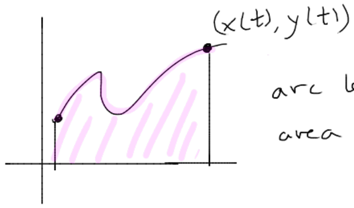
④ sub "this" into $y(t)$

$y(t) = \frac{1}{t}$

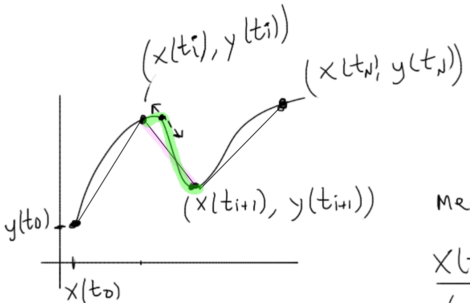
$y(t) = \frac{1}{\left(\frac{x+1}{5}\right)} = \frac{5}{x+1}$

⑤ Finally $y = \frac{5}{x+1}$

has parametric equations



arc length ?
area under curve?



Compute arc length via "calculus"
– approx, refine, repeat

$\delta t_i = t_{i+1} - t_i$

shaded segment : $\sqrt{(x(t_{i+1}) - x(t_i))^2 + (y(t_{i+1}) - y(t_i))^2}$

$= \sqrt{(x'(t_i^*) \cdot \delta t_i)^2 + (y'(t_i^*) \cdot \delta t_i)^2}$

$= \sqrt{(x')^2 \cdot \delta t^2 + (y')^2 \cdot \delta t^2}$

$= \sqrt{\delta t^2 \cdot ((x')^2 + (y')^2)}$

$= \sqrt{x'(t_i^*)^2 + y'(t_i^*)^2} \; \delta t$

length of shaded segment

Mean Value Theorem

$$\frac{x(t_{i+1}) - x(t_i)}{(t_{i+1} - t_i)} = x'(t_i^*) \cdot$$

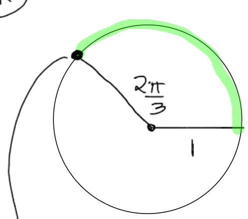
avg rate of change = deriv. @ some pt

$$\text{Total Approx Length} = \sum_{i=1}^{3} \sqrt{x'(t_i^*)^2 + y'(t_i^*)^2} \; \delta t$$

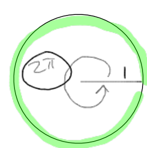
$$\text{Total Exact Length} = \lim_{N \to \infty} \sum_{i=1}^{N} \sqrt{x'(t_i^*)^2 + y'(t_i^*)^2} \; \delta t = \int_{t_0}^{t_N} \sqrt{(x'(t))^2 + (y'(t))^2} \, dt$$

arc length of parametric curve $(x(t), y(t))$

ex



From geometry:
$l = \frac{2\pi}{3}$



$C = 2\pi r = 2\pi$

parameterized by $(\cos t, \sin t)$
$0 \leq t \leq \frac{2\pi}{3}$

$$l = \int_0^{\frac{2\pi}{3}} \sqrt{\underbrace{(-\sin t)^2 + (\cos t)^2}_{\sin^2 + \cos^2 = 1}} \, dt = \int_0^{\frac{2\pi}{3}} 1 \, dt$$

$$= t \Big|_0^{\frac{2\pi}{3}} = \frac{2\pi}{3}$$