$$\frac{dy}{dt} + 2y = e^{-6t} \quad y(0) = 1$$

Find the general solution of the first–order linear differential equation.

$$2xy' - y = x^{-1}$$

Determine the general solution of the first-order linear differential equation.

$$e^x y' = 1 + 7e^x y$$

Solve the Initial Value Problem.

$$y' + 4y = 0,\ y(\ln(3)) = 3$$

Solve the initial value problem.

$$y^{10}\frac{dy}{dx} = x^{-6},\ y(1) = 0$$

Differential Eqn's

(Next week Last topic (Polar Coords))

$y' = \frac{g(x)}{f(y)}$ or $y' = \frac{f(y)}{g(x)}$ ← $f(y)\,dy = g(x)\,dx$

Summary so far:

You'll see two types { separable ⇒ $f(y)\,dy = g(x)\,dx$; linear ⇒ $y' + P(x)\,y = Q(x)$

$y' + P(x)\,y = Q(x)$ ↕ $x + my = b$; $y = -\frac{1}{m}x + \frac{b}{m}$

$y' + 4y = x$

$x^2 y' + 5xy = e^x$; $y' = \frac{e^x - 5xy}{x^2}$

Solve the Initial Value Problem

(ex) $\frac{dy}{dt} + 2y = e^{-6t}$, $y(2) = 5$

sol'n: $y = \left(-\frac{1}{4}\right)e^{-6t} + \left[5 + \frac{1}{4}\left(e^{-12}\right)\right]$

① "linear" : $y' + P(t)y = Q(t)$

① Integrating Factor:

$e^{\int P(t)\,dt} = e^{\int 2\,dt} = e^{2t}$

② multiply D.E. by it

$e^{2t}(y' + 2y) = e^{2t} \cdot e^{-6t}$

③ perceive LHS as derivative of product

$LHS = e^{2t}y' + e^{2t}\cdot 2y = \frac{d}{dx}\left(y \cdot e^{2t}\right)$

product rule $\frac{d}{dt}(FG) = F'G + FG'$

④ $\frac{d}{dt}\left(y \cdot e^{2t}\right) = e^{-4t}$

⑤ $\int \frac{d}{dt}\left(y \cdot e^{2t}\right)dt \quad \int e^{-4t}\,dt$

$y \cdot e^{2t} = e^{-4t}\left(-\frac{1}{4}\right) + C$

⑥ isolate $y = \left(e^{-4t}\left(-\frac{1}{4}\right) + C\right)e^{-2t}$

$y = \left(-\frac{1}{4}\right)e^{-6t} + Ce^{-2t}$ genl sol'n

⑦ use I.V. $y(2) = 5$; sub $t = 2$, sub $y = 5$

$5 = \left(-\frac{1}{4}\right)e^{-12} + Ce^{-4}$

$\left[5 + \frac{1}{4}\left(e^{-12}\right)\right]e^{4} = C$

$\int \frac{d}{dt}\left(y \cdot e^{2t}\right)dt = \int \frac{dy}{dt}\,e^{2t}\,dt = \int e^{2t}\,dy$

$y \cdot e^{2t} =$

(ex) Solve the I.V.P. (particular sol'n)

$$y^{10}\frac{dy}{dx} = x^{-6} \qquad y(1) = 6$$

① Separable: y's on left, x's on right

① $y^{10}\,dy = x^{-6}\,dx$

② $\int y^{10}\,dy = \int x^{-6}\,dx$

$$\frac{y^{11}}{11} = \frac{x^{-5}}{-5} + C$$

↓ gen'l sol'n $y = \ldots$

③ use I.V. $x=1$, $y=0$

$$\frac{0^{11}}{11} = \frac{1^{-5}}{-5} + C \qquad (1^{-5} = 1)$$

$$\frac{1}{5} = C$$

④ put together & isolate y

$$\frac{1}{11}y^{11} = \frac{x^{-5}}{-5} + \frac{1}{5} \qquad \left(-\frac{1}{5}\right)$$

$$y = \left(11\left(\frac{-1}{5x^5} + \frac{1}{5}\right)\right)^{1/11}$$

$$\rightarrow \quad y = \left(\frac{11}{5}\right)^{1/11}\left(\frac{-1}{x^5} + 1\right)$$

part. sol'n