Week 12 Mon

Exam 4 Topics:

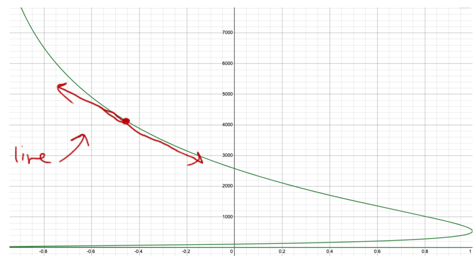
parametric curves : ex $x(t) = \cos(t)$, $y(t) = e^t + 1$

- d/dt, ex $\frac{dx}{dt} = -\sin(t)$, $\frac{dy}{dt} = e^t$

- $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$, ex $\frac{dy}{dx} = \frac{e^t}{-\sin(t)}$

slope of tan line

- plot: $(x(t), y(t))$

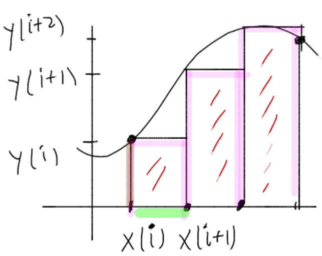


- arc length

ℓ = length of curve parameterized by $(x(t), y(t))$ b/w $t=a$, & $t=b$

$$\ell = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}$$

- Area under parametric curve:



= length (height) × width

$$\text{Approx Area} = \sum_{i=0}^{N} \text{Area of } i\text{th rectangle}$$

$$= \sum_{i=0}^{N} y(i) \cdot (x(i+1) - x(i))$$

$$= \sum_{i=0}^{N} y(i) \cdot \underbrace{(x(i+1) - x(i))}_{=\Delta x} \cdot \frac{t(i+1) - t(i)}{t(i+1) - t(i)} \qquad \frac{\Delta t}{\Delta t}$$

$\Delta y / \Delta x \xrightarrow{\text{lim}}$

$$= \sum_{i=0}^{N} y(i) \cdot \frac{\Delta x}{\Delta t} \cdot \Delta t$$

$$\text{Exact Area} \lim_{N \to \infty} \sum_{i=0}^{N} y(i) \cdot \frac{\Delta x}{\Delta t} \cdot \Delta t = \int_{t(i)}^{t(N)} y(t)\, \underbrace{x'(t)}_{= \frac{dx}{dt}}\, dt$$

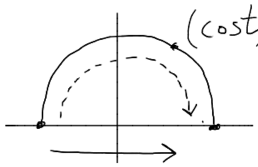
Area: under Parametric Curve

$$\boxed{A = \int_a^b y(t) \cdot x'(t)\, dt}$$

Compute the area of a semi-circle w/ radius $= 1$

$A = \pi r^2$

$\frac{1}{2} A = \frac{\pi (1)^2}{2} = \frac{\pi}{2}$



$(\cos t, \sin(t)) \Big|_{t \in (0, \pi)}$

power reduction:

$\sin(A+B) = \sin A \cos B + \sin B \cos A$

$A = t = B$

$\sin(2t) = 2 \sin t \cos t$

$$A = \int_{\pi}^{0} \sin(t) \cdot (-\sin(t))\, dt = -\int_{\pi}^{0} \sin^2 t = -\int_{\pi}^{0} \frac{1 - \cos(2t)}{2}\, dt$$

$u = 2t$, $du = 2\,dt$

$$= -\int_{\pi}^{0} \frac{1}{2}\, dt + \int_{\pi}^{0} \frac{\cos(2t)}{2}\, dt = -\frac{1}{2} t \Big|_{\pi}^{0} + \frac{1}{4} \sin(2t) \Big|_{\pi}^{0}$$

$$= -\frac{1}{2}(0) - \underbrace{\left(-\frac{1}{2}(\pi)\right)}_{\pi/2} + \frac{1}{4} \underbrace{\sin(0)}_{=0} - \frac{1}{4} \underbrace{\sin(\pi)}_{=0}$$

$$= \pi/2$$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$A = t = B$

$\cos(2t) = \cos^2 t - \sin^2 t$

$= 1 - \sin^2 t - \sin^2 t$

$\frac{1}{2} \int \cos(2t) \frac{2\, dt}{2}$

$\frac{1}{2} \int \cos u \frac{du}{2} = \frac{1}{4} \int \cos u = \frac{1}{4} \sin(u) = \frac{1}{4} \sin(2t)$

ex

Compute area under cycloid from $t=0$ to $t=2\pi$

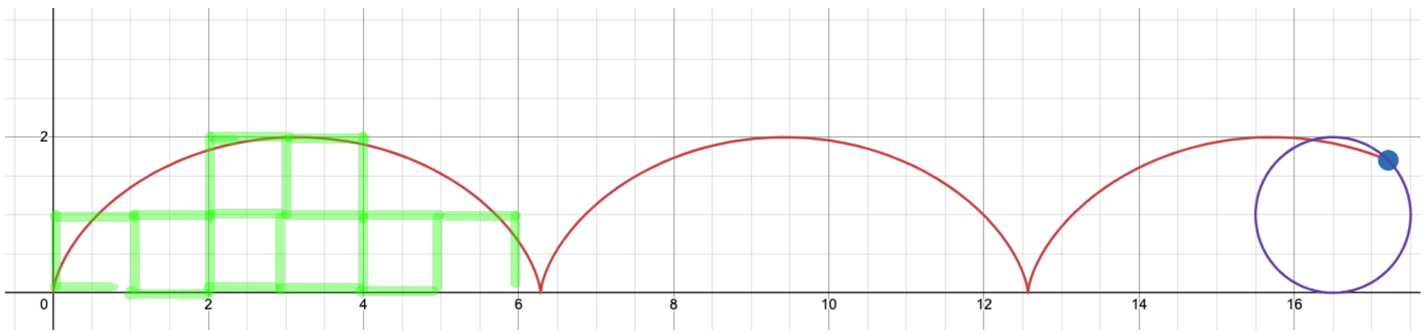
$$\frac{1+\cos(2t)}{2}$$

$x(t) = t - \sin(t)$

$y(t) = 1 - \cos(t)$

$$A = \int_0^{2\pi} (1-\cos t)\underbrace{(1-\cos t)}_{x'(t)}\,dt = \int_0^{2\pi} 1 - 2\cos t + \cos^2 t$$

$$= \int_0^{2\pi} 1\,dt - 2\int_0^{2\pi} \cos t\,dt + \int_0^{2\pi} \frac{1}{2}\,dt + \frac{1}{2}\int_0^{2\pi} \cos(2t)\,dt$$



Surface Area of Revi

$x(t) = 13t - \sin t$

$y(t) = 13 - \cos t$

$$A = \int 2\pi f(x) \cdot \sqrt{1 + [f'(x)]^2}\, dx$$