wk 12

Wed. : Differential Equations

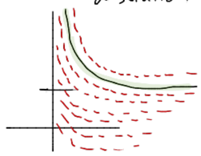
Newton's Law of Cooling

ex $\frac{dT}{dt} = k(A - T)$ — k ↓ constant

A = ambient temp

T = temp @ time t

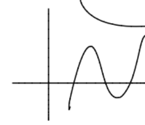
a solution to a D.E. is a function



family of sol'ns comes from the +C

$\int f(x)\,dx = F(x) + C$

alg. eqn



(dy/dx)

ex $\frac{dy}{dx} = 1 - 6e^{2x}$ (directly integrable)

① $dy = 1 - 6e^{2x}\,dx$

② $\int 1\,dy = \int 1 - 6e^{2x}\,dx$

$y = x - 3e^{2x} + C$

ex $\underbrace{y' = yx}_{\text{separable}}$

Common Task: Solve the D.E.

- means: find the (a) function(s) whose derivative satisfies the D.E.
- General Solution: Family (∞ #) of functions that satisfy the D.E.
- Particular Solution: Single sol'n satisfying both D.E. and some additional 'initial conditions'

eg Temp of coffee @ t=0

ex Find the Particular Sol'n to

D.E. — initial cond.

$\frac{dy}{dx} = 1 - 6e^{2x}$, $y(0) = 5$

↑ sub x=0, set y=5

① Find gen'l sol'n: $y = x - 3e^{2x} + C$

② use I.C. ⇒ $5 = 0 - 3e^{2\cdot 0} + C$

⇒ $C = 8$

③ ANS: $y = x - 3e^{2x} + 8$

Separable D.E.'s

Def'n $\frac{dy}{dx} = \underbrace{\text{Function of } x}$ times $\underbrace{\text{Function of } y}$ i.e., $\boxed{\frac{dy}{dx} = f(x)\cdot g(y)}$

↑ mult.

why? B/c using division/mult. you can separate the x's from y's

① $\frac{1}{g(y)}\frac{dy}{dx} = f(x)$

② $\frac{1}{g(y)}\,dy = f(x)\,dx$

y's on left, x's on right

How to Solve? ① separate as above
② integrate both sides (wrt y on left, x on right)

(ex) $y' = yx \rightsquigarrow \frac{dy}{dx} = yx$

① $\frac{y'}{y} = x$

② $\frac{\frac{dy}{dx}}{y} = x$

③ $\frac{dy}{y} = x\,dx$

④ $\int \frac{dy}{y} = \int x\,dx$ $\quad \int \frac{1}{y}\,dy = \ln|y|$

⑤ $\ln|y| = \frac{x^2}{2} + C$

⑥ isolate y

⑦ $e^{\ln|y|} = e^{\left(\frac{x^2}{2} + C\right)}$

⑧ $|y| = e^{\frac{x^2}{2}}\cdot e^{C} = e^{\frac{x^2}{2}}\cdot C = Ce^{x^2/2}$

⑨ $y = Ce^{\frac{x^2}{2}}$ general sol'n

(ex) If $c = 1 \Rightarrow y = e^{x^2/2}$
$y' = e^{x^2/2}\cdot\left(\frac{2x}{2}\right) = \underbrace{e^{\frac{x^2}{2}}}\cdot x$
$= y\cdot x$

(ex) If $c = 5$ $\quad y = 5e^{x^2/2}$

$y' = 5e^{x^2/2}\cdot x = y\cdot x$

Solve the Initial Value Problem: I.V.P.

$y \cdot y' = x e^{-y^2}, \quad y(0) = 5$

separable? get y's on left x's on right : $y \cdot dy \cdot e^{y^2} = x\,dx$

$e^{y^2} y\,dy = x\,dx$

$y = \sqrt{\ln(x^2 + e^{25})}$

$\frac{1}{2}\int e^{y^2} 2 \cdot y\,dy = \int x\,dx$

$u = y^2$
$du = 2y\,dy$

$= \frac{x^2}{2} + C$

$\frac{1}{2}\int e^u du = \frac{1}{2}e^u = \frac{1}{2}e^{y^2}$ ← left side

$\frac{1}{2}e^{y^2} = \frac{x^2}{2} + C$

$e^{y^2} = x^2 + 2C = x^2 + C$ (or just rename)

$\ln(e^{y^2}) = \ln(x^2 + C)$

$y^2 = \ln(x^2 + C$

$y = \pm\sqrt{\ln(x^2 + C)}$ gen'l sol'n

use I.C.

$5 = \pm\sqrt{\ln(0^2 + C)} = \pm\sqrt{\ln C}$

$25 = \ln C \Rightarrow C = e^{25}$