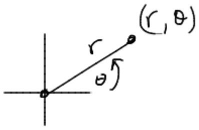
Polar Coordinates

(r, θ) angle

radius



Arc length:

$(r, \theta) = (f(\theta), \theta)$

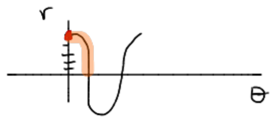
$$\ell = \int_{\theta_1}^{\theta_2} \sqrt{f(\theta)^2 + f'(\theta)^2}\, d\theta$$

Polar Area

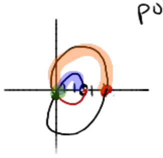
$$A = \int_{\theta_1}^{\theta_2} \frac{1}{2} r^2 d\theta = \int_{\theta_1}^{\theta_2} \frac{1}{2} (f(\theta))^2 d\theta$$

$r = 1 + 4\cos\theta$

think: $y = 1 + 4\cos x$



polar



$(-3, \pi) \leftrightarrow (3, 0)$

θ	0	$\cos^{-1}(-1/4)$	π		$2\pi - \cos^{-1}(-1/4)$	2π
r	5	0	-3	$-$	0	5

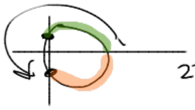
$1 + 4\cos 2\pi$ $\underbrace{\cos 2\pi}_{=1}$

$1 + 4\cos\theta = 0$

$\cos\theta = \frac{-1}{4}$

$\theta = \cos^{-1}(-\frac{1}{4})$

what angle in $[0, \pi]$ gives x-coord $= -1/4$



$2\pi - \cos^{-1}(-1/4)$

ORANGE LENGTH

$$= \int_0^{\cos^{-1}(-1/4)} \sqrt{(1+4\cos\theta)^2 + (-4\sin\theta)^2}\, d\theta$$

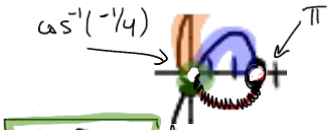
$$= \int_0^{\cos^{-1}(-1/4)} \sqrt{1 + 8\cos\theta + \underbrace{16\cos^2\theta + 16\sin^2\theta}_{16}}$$

$$= \int_0^{\cos^{-1}(-1/4)} \sqrt{17 + 8\cos\theta}\, d\theta$$

evaluate numerically

≈ 8.3

Blue length:



$$= \int_{\cos^{-1}(-1/4)}^{\pi} \sqrt{17 + 8\cos\theta}\, d\theta = -\int_{2\pi - \cos^{-1}(-1/4)}^{\pi} \sqrt{17 + 8\cos\theta}\, d\theta$$

Polar Area

$$A = \int_{\theta_1}^{\theta_2} \frac{1}{2} r^2 \, d\theta = \int_{\theta_1}^{\theta_2} \frac{1}{2} (f(\theta))^2 \, d\theta$$

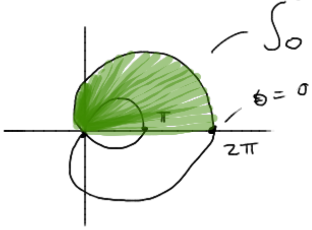
Total Shaded Area

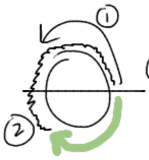
$$\int_0^{\cos^{-1}(-1/4)} 1 + 8\cos\theta + 16\cos^2\theta \, d\theta$$

$$-\frac{1}{2}\int_{-\cos^{-1}(-1/4)}^{\cos^{-1}(-1/4)} 1 + 8\cos\theta + 16\cos^2\theta \, d\theta$$

$r = 1 + 4\cos\theta$

$$\int_0^{\cos^{-1}(-1/4)} \frac{1}{2}(1+4\cos\theta)^2 \, d\theta = \underbrace{\frac{1}{2}\int_0^{\cos^{-1}(-1/4)} 1 + 8\cos\theta + 16\cos^2\theta \, d\theta}_{\text{total area inside} \times 2}$$

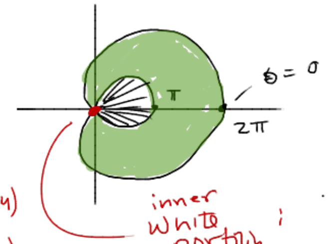




① $\cos^{-1}(-1/4)$

② $2\pi - \cos^{-1}(-1/4)$

③ $-\cos^{-1}(-1/4)$



inner white portion exactly

$$\frac{1}{2}\int_{-\cos^{-1}(-1/4)}^{\cos^{-1}(-1/4)} 1 + 8\cos\theta + 16\cos^2\theta \, d\theta$$