Name:

Math 163	Take-Home Exam 4
Date:	December 1, 2025
Total Points:	100

Numerical

Numerical approximations are allowed. Solutions may be on paper or in Desmos, python, R, etc. (You may use $f'(x)$ notation in Desmos.)

1. In the following consider the polar curve $r = 3\cos(6\theta) + 1, 0 < \theta < \pi$.
   (1.1) Sketch a graph of the polar curve.
   (1.2) Find the arc length of the curve.

2.(2.1) Find the area bound by all loops of the polar curve

$$r = 6\cos(\theta) + 6\cos(3\theta) + 6\cos(5\theta) - 6\cos(7\theta) - 6\cos(9\theta)$$

   (2.2) Plot the circle whose area differs from the area above by at most 10^{-2}.
   (2.3) If appropriate compute the center of mass of the polar curve.
   (2.4) Draw the circle with radius above at the center of mass of the polar curve, recognizing that most of the curve lies within it.

3. Find the surface area of the torus obtained by rotating the circle $x^2 + (y-14)^2 = 49$ about the x-axis.

4. Find the two inflection points (remember where $\frac{dy}{dx} = 0$) of the limacon $r = 4\cos(\theta) - 6$.

Exact

Solutions are to be exact, and completely on paper. No numerical approximations.

5. Find the area of region in the figure $r = 8\sin(5\theta)$

6. Find the surface area of the solid obtained by revolving the curve

$$y = \sin(3t), x = \cos(3t)$$

about the x-axis.

7. Solve the following initial value problem:

$$\frac{dy}{dx} = \frac{\sin x}{\cos y}, y(0) = \pi$$

8. Solve the following initial value problem:

$$\frac{dy}{dx} = \frac{(y^2+1)\sec^2(x)}{y}, y(0) = 0$$

9. Solve the following initial value problem:

$$y' - 4y = 8x, y(0) = 5$$

10. Solve the following initial value problem:

$$y' - \frac{3}{x}y = 4, y(1) = 7$$