MA1403 Fri

$$\int e^{12x}\tan^2(e^{12x})\,dx = \int e^{12x}\tan^2(u)\frac{1}{12e^{12x}}du = \frac{1}{12}\int \tan^2(u)\,du$$

① u-sub to simplify

$u = e^{12x}$ | $dx = \frac{1}{12e^{12x}}du$

$du = 12e^{12x}dx$

② Pythagorean ⇒ $\tan^2 u = \sec^2 u - 1$

check!

$$\frac{d}{dx}(\text{ans}) = \frac{1}{12}\left[\sec^2(e^{12x})\cdot e^{12x}\cdot 12 - 12e^{12x}\right]$$

d/dx

$$= e^{12x}\left[\sec^2(e^{12x}) - 1\right]$$

$$= e^{12x}\tan^2(e^{12x})$$

:)

③ $\frac{1}{12}\int \sec^2(u) - 1\,du = \frac{1}{12}\int \sec^2(u) - \frac{1}{12}\int 1\,du = \frac{1}{12}\tan(u) - \frac{1}{12}\cdot u + C$

④ $\frac{1}{12}\left[\tan(e^{12x}) - e^{12x}\right]$

Integrals of the form

These integrals, the method of solving, depends on even / odd - ness of n and m

$$\int \sec^n(x)\tan^m(x)\,dx$$

Ex (special)

$$\int \sec(x)\,dx = \int \sec(x)\cdot 1\,dx = \int \sec(x)\cdot \frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}\,dx$$

$$= \int \frac{\sec^2(x)+\sec(x)\tan(x)}{\sec(x)+\tan(x)}\,dx$$

$u = \sec x + \tan x$

$du = \sec^2(x) + \sec(x)\tan(x)\ dx$

$$= \int \frac{du}{u} = \ln|u| + C = \boxed{\ln|\sec x + \tan x| + C}$$

Note: the sec(x) function is used to create maps of earth odd secant power

$$\underbrace{\int \sec^3(x)\,dx}_{A} = \int \sec^2(x)\sec(x)\,dx = \int (\tan^2(x)+1)\sec(x)\,dx$$

$$= \underbrace{\int \tan^2(x)\sec(x)\,dx} + \underbrace{\int \sec(x)\,dx}_{\underbrace{\ln|\sec(x)+\tan(x)|}_{Y}} =$$

$$= \int \underbrace{\tan x}_{u}\,\underbrace{\sec x\tan x\,dx}_{dv}$$

Idea: Find A

$A = X - A + Y$

$2A = X + Y$

$A = \frac{X+Y}{2}$

$u = \tan x \quad dv = \sec x\tan x\,dx$

$du = \sec^2 x \quad v = \sec x$

$$= \underbrace{\sec x\tan x}_{X} - \int \underbrace{\sec^3(x)}_{A}\,dx$$

now add $\int \sec^3 x$ to both sides

$$2\int \sec^3(x)\,dx = \sec x\tan x + \ln|\sec x+\tan x|$$

) $\div$ by 2

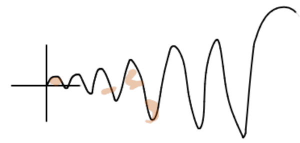
$$\int \sec^3(x)\,dx = \frac{1}{2}\left[\sec x\tan x + \ln|\sec x+\tan x|\right]$$

Similar idea works here:

$$\int e^x \cos x \, dx$$

LIPET

LIATE



It's a product, so I.B.P.

$u = e^x \quad dv = \cos x \, dx$

$du = e^x dx \quad v = \sin x$

$u = e^x \quad dv = \sin x \, dx$

$du = e^x dx \quad v = -\cos x$

similar, but not exactly

$$\int e^x \cos x \, dx = e^x \sin x - \int e^x \sin x \, dx = e^x \sin x \overset{+}{-} \left[e^x(-\cos x) - \int e^x (-\cos x) dx \right]$$

$$= e^x \sin x + e^x \cos x - \underbrace{\int e^x \cos x \, dx}_{\text{goal}}$$

add $\int e^x \cos x dx$

$$2 \int e^x \cos x \, dx = e^x(\sin x + \cos x)$$

$$\Rightarrow \quad \int e^x \cos x \, dx = 2 e^x (\sin x + \cos x)$$

$\int \sec^2 x \tan^2 x \, dx$ — even power of tan

$\Rightarrow$ pull off one tan, combine w/ one sec to get $\sec x \tan x$

this is a good du

$\int \sec x \tan x \sec x \tan \, dx$ ★

$u = \sec x \tan x \qquad dv = \sec x \tan x$

$u = \sec$

easier: u-sub

$u = \tan x$
$du = \sec^2 x \, dx$

$= \int u^2 \, du = \frac{u^3}{3} + C = \frac{\tan^3 x}{3} + C$

If $\int \sec^4 x \tan^2 \, dx$ then ★ works best

Next Week
Trig. Sub

$u = \ln x$

$du = \frac{1}{x}$

$$\int \frac{\tan^3(\ln x)}{6x}$$

$$\frac{1}{6}\int \tan^3(u)\, du$$

$$\tan^2(u)\, \tan(u)$$

$$\frac{1}{6}\int (\sec^2 u - 1)\tan x$$

$$\frac{1}{6}\int \sec^2 u \tan u - \frac{1}{6}\int \tan u$$

$$\frac{1}{12}\tan^2(u) + \frac{1}{6}\ln|\cos|\ln x||$$

$$\int \frac{\sin}{\cos} \quad \frac{du}{u} = -\ln|\cos u|$$

$u = \cos$

$$15 \int (\sin^{-1} x)\, dx = x\sin^{-1}x - 2\int \sin^{-1}x \underbrace{\left(\frac{x}{\sqrt{1-x^2}}\right) dx}$$

$$-\frac{1}{2}\int (1-x^2)^{-1/2} -2x$$

$u = 1-x^2$

$du = -2x$

$u = (\sin^{-1})^2 \qquad dv = dx$

$du = 2\sin^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) \qquad = x$

$u = \sin^{-1} \qquad dv = \underbrace{\frac{x}{\sqrt{1-x^2}}}$

$du = \frac{1}{\sqrt{1-x^2}} \qquad v = -\frac{1}{2}(1-x^2)^{1/2}$

$= \sin^{-1}x - \frac{1}{2}(1-x^2)^{1/2} - \underbrace{\int -\frac{1}{2}\, dx}_{+\frac{1}{2}x}$

① IBP $u = (\sin^{-1})^2$

② another IBP $u = \sin^{-1}$

③ things work out