MA163 thur

$$\int 18\left(\sin^{-1}(x)\right)^2 dx$$

First: I.B.P $\rightarrow$ LIPET

$$\int \sin^{-1}(x)\,dx$$

EVERYTHING in the integral gets used in I.B.P.

$u = \sin^{-1}x \qquad dv = dx$

$du = \dfrac{1}{\sqrt{1-x^2}}\,dx \qquad v = x$

$$= x\cdot\sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}}\,dx$$

$$= x\sin^{-1}x - \int (1-x^2)^{-\frac{1}{2}}\underbrace{x\,dx}_{\text{almost exactly } du}$$

$u = 1 - x^2$

$du = -2x\,dx$ | $dx = -\dfrac{1}{2x}dx$

$$= x\sin^{-1}x - \frac{1}{-2}\int (\underbrace{1-x^2}_{u})^{-\frac{1}{2}}\underbrace{(-2)x\,dx}_{du}$$

$$= x\sin^{-1}x + \frac{1}{2}\int u^{-1/2}\,du$$

$$= x\sin^{-1}x + \frac{1}{2}\cdot\frac{u^{1/2}}{1/2} + c = x\sin^{-1}x + (1-x^2)^{\frac{1}{2}} + c$$

Integrals of Form $\int \sin^n(x)\cos^m(x)\,dx$

(*)

$$\int \sin(x)\cos(x)\,dx = \int u\,du = \frac{u^2}{2} + c$$

(u-sub)

$$= \boxed{\frac{(\sin x)^2}{2} + c}$$

$u = \sin x$

$du = \cos x\,dx$

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these sols are equal since

$\cos(x+x) = \cos^2 x - \sin^2 x$

$\cos(2x) = (1-\sin^2 x) - \sin^2 x = 1 - 2\sin^2 x$

$-\frac{1}{4}\cos(2x) = -\frac{1}{4} + \frac{1}{2}\sin^2 x$

$$\boxed{\sin^2 x = \frac{1-\cos 2x}{2}}$$

power reduction or ½ angle

(trig sub ... this method works in many other problems too)

$\sin(x+y) = \sin x\cos y + \sin y\cos x$

so if $x = y$

$$\boxed{\sin(2x) = 2\sin x\cos x}$$

use!

$u = 2x \quad du = 2dx$

$(*) = \frac{1}{2}\int \frac{1}{2}\sin(2x)\,2\,dx$

$= \frac{1}{4}\int \sin(u)\,du = \boxed{-\frac{1}{4}\cos(2x) + c}$

Ex

$$\int \sin^5(x)\cos^2(x)\,dx = \int \sin^4(x)\cos^2(x)\underbrace{\sin(x)\,dx}_{du}$$

$$\boxed{-\frac{\cos^3 x}{3} + \frac{2\cos^5 x}{5} - \frac{\cos^7 x}{7} + C}$$

Key: ① look for odd power

② separate off one power, & put @ end (this one term becomes du)

③ think $u = \cos(x)$

④ use trig ID's to replace $\sin^4(x)$ (even power) w/ $\cos(x)$

$$\sin^4(x) = (\sin^2 x)^2 = (1-\cos^2 x)^2 = 1 - 2\cos^2 x + \cos^4 x$$

⑤ $= \int (1 - 2\cos^2 x + \cos^4 x)\cos^2 x \sin x \, dx$

$u = \cos x$

$du = -\sin x \, dx$ | $dx = \frac{1}{-\sin x} du$

⑥ $= \int (1 - 2u^2 + u^4) u^2 \sin x \left(\frac{1}{-\sin x}\right) du = -\int u^2 - 2u^4 + u^6 \, du = -\frac{u^3}{3} + \frac{2u^5}{5} - \frac{u^7}{7} + C$

Ex (no odd power)

$A^2 \cdot B^2 = (AB)^2$

$$\int \sin^2 x \cos^2 x\,dx = \int (\sin x \cos x)^2 dx$$

$$= \int \left(\tfrac{1}{2}\sin 2x\right)^2 dx$$

distribute power, pull out const

$$= \tfrac{1}{4}\int \sin^2 2x\,dx$$

need to reduce power

b/c $\boxed{\sin^2 x = \frac{1-\cos 2x}{2}}$

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$$= \tfrac{1}{4}\int \tfrac{1}{2}(1-\cos 4x)\,dx$$

$u = 4x$
$du = 4\,dx$

$$= \tfrac{1}{8}\int 1\,dx - \tfrac{1}{8}\int \cos 4x\,dx$$

$\frac{1}{4}$ ↑ ↑ 4

$$= \frac{x}{8} - \frac{1}{32}\int \cos u\,du = \frac{x}{8} - \frac{\sin 4x}{32} + C$$

Ex Similar to:

$$\int \sin^4 x \cos^2 x\,dx = \int \sin^2 x \underbrace{\sin^2 x \cos^2 x}\,dx$$

$(\sin x \cos x)^2 = \left(\tfrac{1}{2}\sin 2x\right)^2$

$= \tfrac{1}{4}\sin^2 2x$

$$= \tfrac{1}{4}\int \underbrace{\sin^2 x}_{} \cdot \underbrace{\sin^2 2x}_{}\,dx$$

(mixed argument)

or relate to $\sin 2x$ or $\cos 2x$

relate to $\sin x$ or $\cos x$

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$$= \tfrac{1}{4}\int \tfrac{1}{2}(1-\cos(2x))\sin^2(2x)\,dx$$

algebra

$$= \tfrac{1}{8}\int \underbrace{\sin^2(2x)\,dx} - \tfrac{1}{8}\int \underbrace{\sin^2(2x)\cdot\cos(2x)\,dx}$$

$\tfrac{1}{8}\int \tfrac{1}{2}(1-\cos(4x))\,dx$

$u = \sin(2x)$
$du = -\cos(2x)\cdot 2\,dx$

$\tfrac{1}{16}\int 1\,dx - \tfrac{1}{16}\int \cos 4x\,dx$

$= -\tfrac{1}{16}\int u^2\,du = \frac{-1}{16}\frac{u^3}{3}$

$u = 4x$
$du = 4\,dx$

$$\tfrac{1}{16}x - \tfrac{1}{64}\sin 4x - \tfrac{1}{16}\frac{\sin^3(2x)}{3} + C$$