#7

$$\int \frac{\tan^3(\ln(x))}{5x}\,dx$$

$\boxed{u = \ln x}$

$du = \frac{1}{x}dx$ } good idea b/c $\frac{1}{x}$ is present

$\boxed{x\,du = dx}$

① u-sub
② trig-sub
③ broke into smaller u-sub

$$= \frac{1}{5}\int \frac{\tan^3(u)}{x}\,x\,du = \frac{1}{5}\int \tan^3(u)\,du = \frac{1}{5}\int \tan(u)\tan^2(u)\,du$$

odd!

$$\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$$

$$\tan^2 + 1 = \sec^2 u$$

u — du relations

$$= \frac{1}{5}\int \tan(u)(\sec^2 u - 1)\,du$$

$dv = -\sin(u)\,du$

$v = \cos(u)$

$$\int \frac{\sin(u)}{\cos(u)}du = \int \frac{-dv}{v} = -\int \frac{1}{v}dv$$

$$= -\ln|v| = -\ln|\cos(u)|$$

$$= -\ln|\cos(\ln(x))|$$

$$= \frac{1}{5}\int \tan(u)\sec^2(u) + \tan(u)\,du$$

good u-sub key! recognize derivative relationships

$w = \tan(u)$

$dw = \sec^2(u)\,du \rightarrow du = \frac{1}{\sec^2(u)}dw$

$$= \frac{1}{5}\int w\,dw = \frac{1}{5}\left[\frac{w^2}{2}\right] = \frac{1}{5}\frac{\tan^2(u)}{2} = \frac{1}{10}\tan^2(\ln(x))$$

Final =

$$= \frac{1}{10}\tan^2(\ln(x)) - \frac{1}{5}\ln|\cos(\ln(x))| + C$$

$$= \frac{1}{5}\int \tan(u)\sec^2(u)\,du - \frac{1}{5}\int \tan(u)\,du$$

$$= \frac{1}{5}\int \left[w\sec^2(u) + w\right]\frac{1}{\sec^2(u)}dw$$

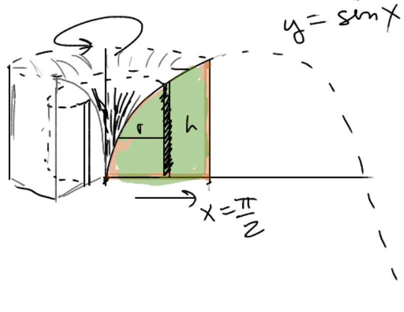
$$= \frac{1}{5}\int w + \frac{w}{\sec^2(u)}dw$$

mixed! :(

Wed wk 2

Application: I.B.P. is used to find volume of some solids of revolution.

Ex.



Shell Method: surface area of shell

$V = 2\pi x f(x)$

given: $f(x) = \sin x$

$$Vol = \int_0^{\frac{\pi}{2}} 2\pi x \cdot \sin x \, dx$$

this is an I.B.P integral.

want: derivative to be simpler

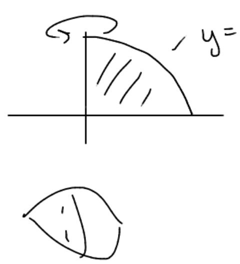
$u = x \qquad dv = \sin x$

$du = dx \qquad v = -\cos x$

$$= 2\pi \int_0^{2\pi} x \cdot \sin x \, dx$$

$$= 2\pi \left[-x\cos x + \underbrace{\int \cos x \, dx}_{\sin x} \right] \Bigg|_0^{\pi/2} = 2\pi \left[-\frac{\pi}{2} \cdot \underbrace{\cos\left(\frac{\pi}{2}\right)}_{0 = x\text{-coord @ } 90^\circ} + \underbrace{\sin\frac{\pi}{2}}_{y\text{-coord @ } 90^\circ = 1} - \left(\overset{0}{\overset{\shortparallel}{0 \cdot \cos 0}} + \overset{0}{\overset{\shortparallel}{\sin 0}} \right) \right] = 2\pi$$

Ex



(this vol. calculation involves a u-sub, then I.B.P.

TRIG INTEGRALS

Main Trig ID's

Pythagorean

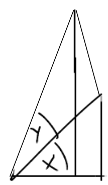
- $\sin^2\theta + \cos^2\theta = 1$
-
-

Trig Sum Formula

$\sin(x+y) = \sin x \cos y + \sin y \cos x$

set $x=y$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$



Other Common

set $x=y$ $\sin(2x) = 2\sin x \cos x$

$\cos(2x) = \cos^2 x - \sin^2 x$

use Pythag

$\cos(2x) = 1 - 2\sin^2 x$

isolate $\sin^2 x$

$\sin^2 x = \dfrac{1-\cos 2x}{2}$

power reduction Formula

$\cos^2 x = \dfrac{1+\cos 2x}{2}$

Integrals of form $\int \sin^n(x)\cos^m(x)\,dx$

($n=m=1$ we did this earlier today u-sub

Ex $\int \sin^4(x)\cos^3(x)\,dx = \int \sin^4 x\,\cos^2(x)\cos(x)\,dx$

1. identify the odd power (if both are even try something else)
2. peel of a square from it
3. use the pythagorean to replace the square
4. set u = the base of the even power .... the odd power base become du

$= \int \sin^4 x\left(1-\sin^2 x\right)\cos(x)\,dx$

$u = \sin x$

$du = \cos x\,dx$

$= \int u^4(1-u^2)\,du = \int u^4 - u^6\,du = \frac{u^5}{5} - \frac{u^7}{7} + C$

$= \frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + C$:)