

WK 3 - Fri

Note: Study guide

$$\int \frac{(5x-3)dx}{x^2-13x+42} = \int \frac{5x-3}{(x-6)(x-7)} dx$$

(partial factors) factor ①

② distinct factors $\Rightarrow \int \frac{A}{x-6} + \frac{B}{x-7} dx$

③ clear denom $A(x-7) + B(x-6) = 5x-3$

④ sub: $x=7 \Rightarrow A \cdot 0 + B(1) = 5 \cdot 7 - 3$
 $B = 32$

$x=6 \Rightarrow A(6-7) = 5 \cdot 6 - 3$
 $-A = 27 \Rightarrow A = -27$

⑤ $= \int \frac{-27}{x-6} + \frac{32}{x-7} dx = -27 \ln|x-6| + 32 \ln|x-7| + C$

if $5(2x-13)$
 $\int \frac{(10x-65)}{x^2-13x+42} dx$
key: derivative relations
 $\int \frac{2x-13}{x^2-13x+42} dx$

ez: u-sub

$$u = x^2 - 13x + 42$$

$$du = (2x - 13) dx$$

$$\frac{1}{2x-13} du = dx$$

$$\int \frac{5(2x-13)}{u} \cdot \frac{1}{2x-13} du$$

$$= \int \frac{5}{u} du = 5 \ln|x^2-13x+42| + C$$

$$\int \frac{14}{(x-4)^2(x-1)} dx \Rightarrow \frac{14}{\cancel{(x-4)^2(x-1)}} \stackrel{\text{from algebra}}{=} \frac{A}{x-1} + \frac{B}{(x-4)} + \frac{C}{(x-4)^2}$$

⑤: not u-sub,

① partial Fraction $\Rightarrow$ repeated root

② clear denom: $14 = A(x-4)^2 + B(x-1)(x-4) + C(x-1)$
 mult. both sides by $(x-4)^2(x-1)$

③ sub: $x=1 \Rightarrow 14 = A(-3)^2 + B \cdot 0 + C \cdot 0 = 9A, \quad A = \frac{14}{9}$

$x=4 \Rightarrow 14 = A \cdot 0 + B \cdot 0 + C(3) \Rightarrow C = \frac{14}{3}$

④ update eq'n using A, C: $14 = \frac{14}{9}(x-4)^2 + B(x-1)(x-4) + \frac{14}{3}(x-1)$

⑤ plug in some x-value, (not $x=1, x=4$)
 these won't help finding B : $x=0$

$$14 = \frac{14}{9}(-4)^2 + B(-1)(-4) + \frac{14}{3}(-1)$$

$$14 = \frac{14 \cdot 16}{9} + 4B - \frac{14}{3} \quad \downarrow \times 9$$

$$9 \cdot 14 = 14 \cdot 16 + 36B - 14 \cdot 3$$

$$-\frac{14}{9} = 14 \left(\frac{9 - 16 + 3}{36} \right) - \frac{9 \cdot 14 - 14 \cdot 16 + 14 \cdot 3}{36} = B$$

$$\textcircled{6} = \int \frac{14/9}{x-1} - \frac{14/9}{x-4} + \frac{14/3}{(x-4)^2} dx = \frac{14}{9} \ln|x-1| - \frac{14}{9} \ln|x-4| - \frac{14}{3(x-4)} + C$$

$$\frac{14}{3} \int \frac{1}{(x-4)^2} dx = \frac{14}{3} \int u^{-2} du = -\frac{14}{3} u^{-1} \rightarrow$$

$u = x-4$
 $du = dx$

Work them, alone w/o notes, give yourself ≈ 1 hour.

Math 163 - Calculus - Exam 1 - Guide

Name: _____

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On the exam you must show your work to receive full credit.

degree = 4 $\Rightarrow$ 4 x I.B.P.
1.

$$\int x^4 e^{3x} dx =$$

sign	u	dv
+	x^4	e^{3x}
-	$4x^3$	$\frac{1}{3}e^{3x}$
+	$12x^2$	$\frac{1}{9}e^{3x}$
-	$24x$	$\frac{1}{27}e^{3x}$
+	24	$\frac{1}{81}e^{3x}$
	$\emptyset$	$\frac{1}{243}e^{3x}$

$$x^4 \cdot \frac{1}{3}e^{3x} - 4x^3 \left(\frac{1}{9}e^{3x} \right) + 12x^2 \left(\frac{1}{27}e^{3x} \right) - 24x \left(\frac{1}{81}e^{3x} \right) + 24 \left(\frac{1}{243} \right) e^{3x}$$

$\oplus uv - \int v du$
 $\oplus (uv - \int v du)$
 $-(uv - \int v du)$

2.

$$\int \frac{4x-1}{x^2-5x-14} dx =$$

3.

$$\int x^4 \sec^2(x^5) dx =$$