HW

$$\int 18\arcsin^2(x)\,dx = 18\int \arcsin^2(x)\,dx$$

$u = \arcsin^2(x)$ $\quad dv = dx$

$du = 2\arcsin(x)\cdot\frac{1}{\sqrt{1-x^2}}dx$ $\quad v = x$

power chain rule

$$= 18\left[x\cdot\arcsin^2(x) - 2\int \arcsin(x)\cdot\frac{1}{\sqrt{1-x^2}}\cdot x\,dx\right]$$

could

$\arcsin(x) = \sin^{-1}(x) \neq \frac{1}{\sin(x)}$

$u = x$ $\quad dv = \arcsin x\cdot\frac{1}{\sqrt{1-x^2}}dx$

$du = dx$ $\quad v = \frac{\arcsin^2(x)}{2}$

$= x\frac{\arcsin^2 x}{2} - \underbrace{\int \frac{\arcsin^2 x}{2}dx}$

this is the O.G. goal, add to both sides.

eg. $\int e^x\cos x\,dx$

$\int \sec^3 x$

$u = \arcsin(x)dx$ $\quad dv = \frac{x}{\sqrt{1-x^2}}dx$

$du = \frac{1}{\sqrt{1-x^2}}dx$ $\quad v = -\sqrt{1-x^2}$

$u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2x}du = dx$

$\int \frac{1}{\sqrt{u}}du$

$$\int dv = \int \frac{x}{\sqrt{1-x^2}}dx = \int \frac{\cancel{x}}{\sqrt{1-x^2}}\cdot\frac{-1}{2\cancel{x}}du$$

$$= -\frac{1}{2}\int u^{-1/2}du$$

$$= -\frac{1}{2}\frac{u^{1/2}}{1/2}$$

$$= -\sqrt{1-x^2}$$

$$= 18x\cdot\arcsin^2 x - 36\left[-\sqrt{1-x^2}\cdot\arcsin x - \int(-1)\,dx\right]$$

$$= 18x\arcsin^2 x + 36\left[\sqrt{1-x^2}\cdot\arcsin x + x\right] + C$$

TRIG SUBSTITUTION: (not trig ID's)

$$\int \cos^2(x)\,dx \overset{\text{trig ID}}{=} \int \frac{1+\cos(2x)}{2}\,dx$$

$$= \int \frac{1}{2}\,dx + \frac{1}{2}\int \cos(2x)\,dx = \frac{1}{2}\cdot\frac{1}{2}\int \cos(2x)\,2dx$$

$u = 2x \quad du = 2dx$

$$= \frac{1}{4}\int \cos u\,du$$

$$= \boxed{\frac{1}{2}x + \frac{1}{4}\sin(2x) + C}$$

radius = 1

Area of circle computation:

 $= 2 \times$  $=$ 

area under curve

$$\int_{-1}^{1} \sqrt{1-x^2}\,dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \underbrace{\sqrt{\cos^2\theta}}_{=\cos\theta} \cdot \cos\theta\,d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta\,d\theta = \frac{1}{2}\theta + \frac{1}{4}\sin(2\theta)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{1}{2}\cdot\frac{\pi}{2} + \frac{1}{4}\underbrace{\sin\left(2\cdot\frac{\pi}{2}\right)}_{\sin(\pi) = 0} - \left[\frac{1}{2}\left(\frac{-\pi}{2}\right) + \frac{1}{4}\underbrace{\sin\left(2\cdot\frac{-\pi}{2}\right)}_{=0}\right]$$

$x = -1 = \sin\theta$

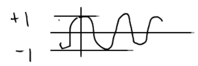
$$= \frac{\pi}{4} + \frac{\pi}{4} = \boxed{\frac{\pi}{2}}$$ — double to get area of circle!

area of upper Half of circle w/ radius = 1

$A = \pi r^2 = \pi$

key: look @ domain: (what are possible x - values)

what trig function gives these exact x - values.



$y = \sin x$

want: y-values of trig fcn = x - vals of integral

let $\theta \in [\frac{-\pi}{2}, \frac{\pi}{2}]$

① $x = \sin(\theta)$

$\frac{dx}{d\theta} = \cos(\theta)$

$dx = \cos\theta\,d\theta \Rightarrow \frac{1}{\cos\theta}dx = d\theta$

② replace $1-x^2$ w/ θ's

$x^2 = \sin^2\theta$

$1-x^2 = 1-\sin^2\theta = \cos^2\theta$

TRIG SUBSTITUTION IS USEFUL FOR ____________

$\int \sqrt{1-x^2}\,dx$, $\int \sqrt{a^2-x^2}\,dx$, $\int \frac{1}{\sqrt{a^2-x^2}}\,dx$, $\int \frac{1}{(a^2-x^2)^{3/2}}\,dx$, ...

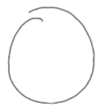
$x = \sin\theta$

$[-1, 1]$

constant minus square under radical

domain

$[-a, a]$



$x = a \sin x$

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fraction, constant plus square …. no radical

$\int \frac{1}{1+x^2}\,dx$,

$\int \frac{1}{a^2+x^2}$ → $x = a\tan\theta$

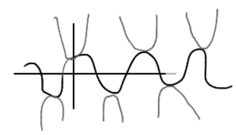
possible vals for x : $\mathbb{R}$

what trig function has range $= \mathbb{R}$? $x = \tan\theta$

$\int \frac{1}{\sqrt{x^2-a^2}}\,dx$

$x = a \cdot \sec\theta$

square minus constant under radical



cos/